Measurement of Coherence Properties of Light

S.V.U. Vedhanth

Introduction – 1st Order

- In physics wherever we studied about light, we assumed it to be monochromatic.

$$\mathbf{E} = \mathbf{E_o} \exp(-\mathbf{i}\boldsymbol{\omega}\mathbf{t} + \boldsymbol{\varphi})$$

where E_o is the amplitude and the ω is the frequency and φ is the phase

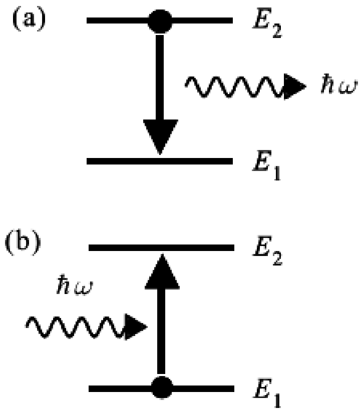

Fig. 4.1 Optical transitions between two states in an atom: (a) spontaneous emission, (b) absorption.

4.1 Einstein coefficients

The quantum theory of radiation assumes that light is emitted or absorbed whenever an atom makes a jump between two quantum states. These two processes are illustrated in Fig. 4.1. Absorption occurs when the atom jumps to a higher level, while emission corresponds to the process in which a photon is emitted as the atom drops down to a lower level. Conservation of energy requires that the angular frequency ω of the photon satisfies:

$$\hbar\omega = E_2 - E_1, \quad (4.1)$$

where E_2 is the energy of the upper level and E_1 is the energy of the lower level. In Section 4.2 we explain how quantum mechanics enables us to calculate the emission and absorption rates. At this stage we restrict ourselves to a phenomenological analysis based on the **Einstein coefficients** for the transition.

The radiative process by which an electron in an upper level drops to a lower level as shown in Fig. 4.1(a) is called **spontaneous emission**. This is because the atoms in the excited state have a natural (i.e. spontaneous) tendency to de-excite and lose their excess energy. Each type of atom

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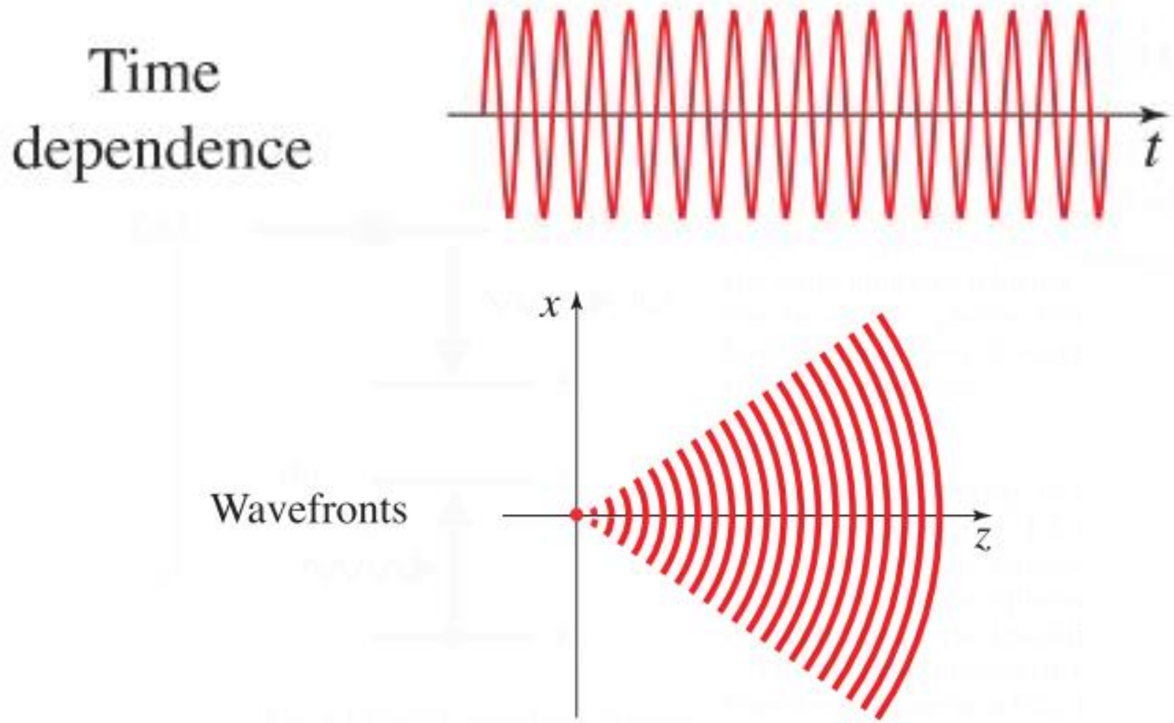

Introduction – 1st Order

But can we have a monochromatic source of light?

It turns out we can't…!

Introduction – 1st Order

- From time dependent perturbation theory, we get an uncertainty relation,

$$\mathbf{\Delta E.\Delta t \approx h/2\pi}$$

- As a result, for $\mathbf{\Delta E} = 0$ we should have $\mathbf{\Delta t} \rightarrow \infty$. It means it may take infinite amount of time for transition.

- So in nature we always have a finite spread in the energy/frequency.

Source	$\Delta\nu_c$ (Hz)
Filtered sunlight ($\lambda_o = 0.4$–$0.8\ \mu$m)	3.74×10^{14}
Light-emitting diode ($\lambda_o = 1\ \mu$m, $\Delta\lambda_o = 50$ nm)	1.5×10^{13}
Low-pressure sodium lamp	5×10^{11}
Multimode He–Ne laser ($\lambda_o = 633$ nm)	1.5×10^{9}
Single-mode He–Ne laser ($\lambda_o = 633$ nm)	1×10^{6}

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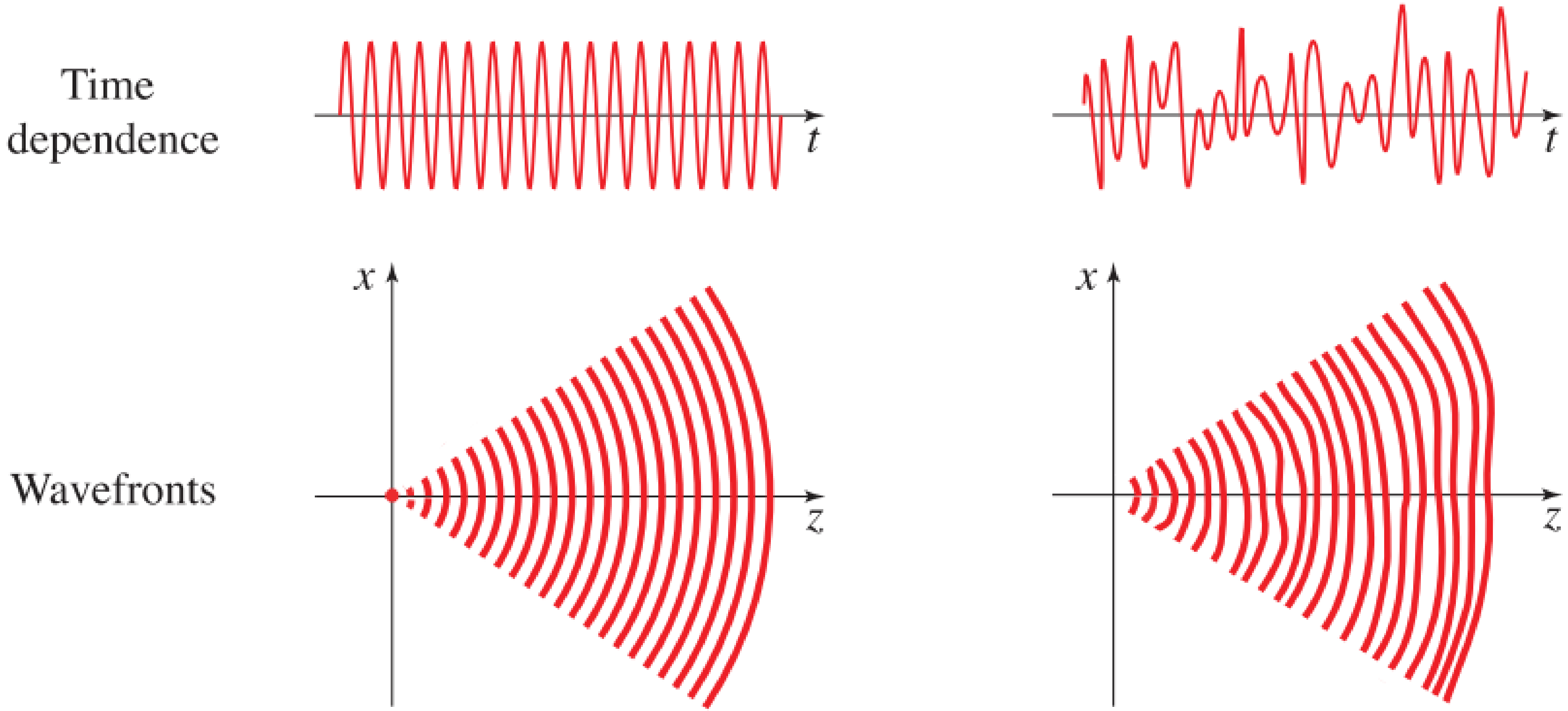
Introduction – 1st Order

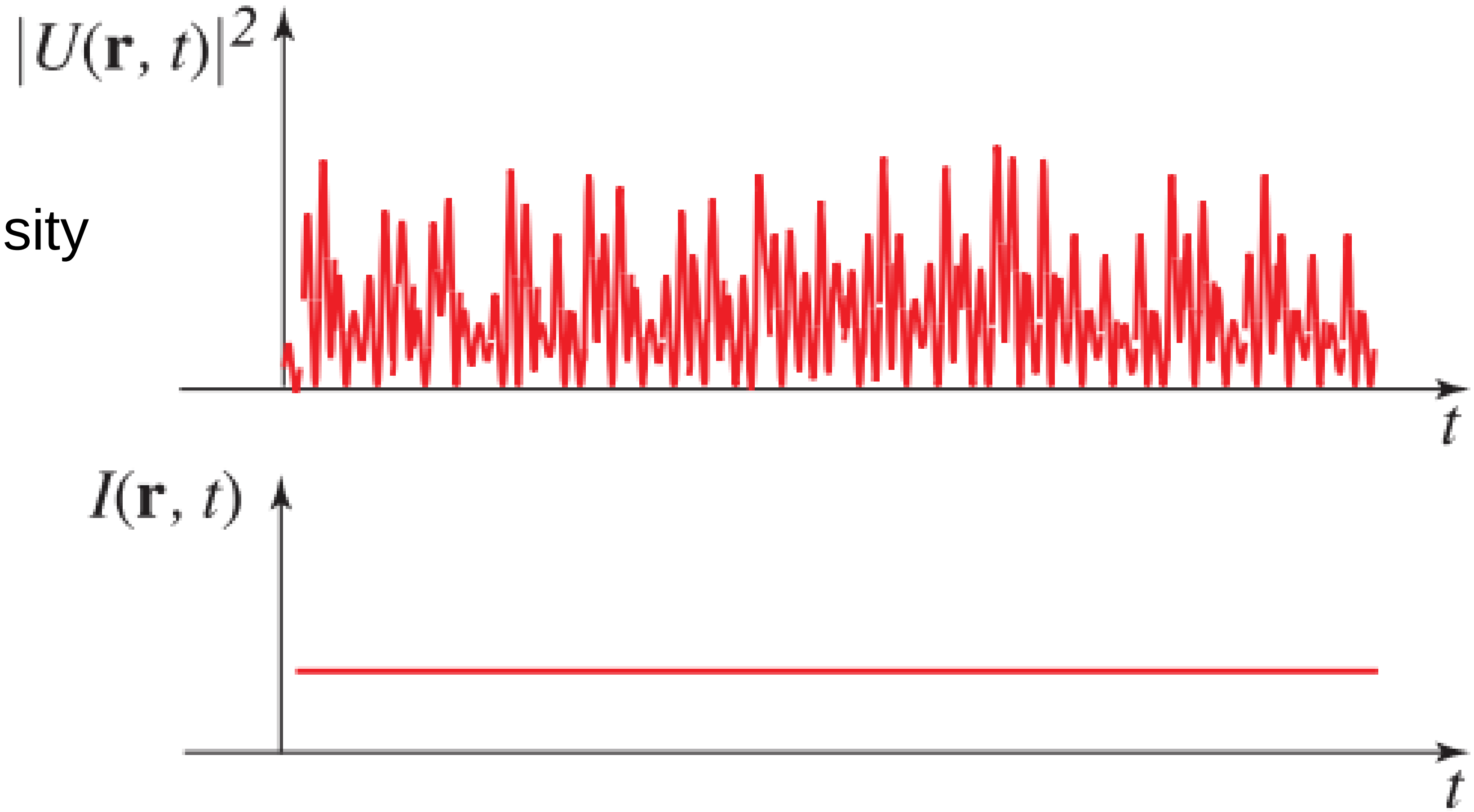
Monochromatic | Random



Fundamentals of Photonics, Saleh and Teich

Introduction – 1st Order

Instantaneous Intensity



Average Intensity

$I = \langle |U(r,t)|^2 \rangle$

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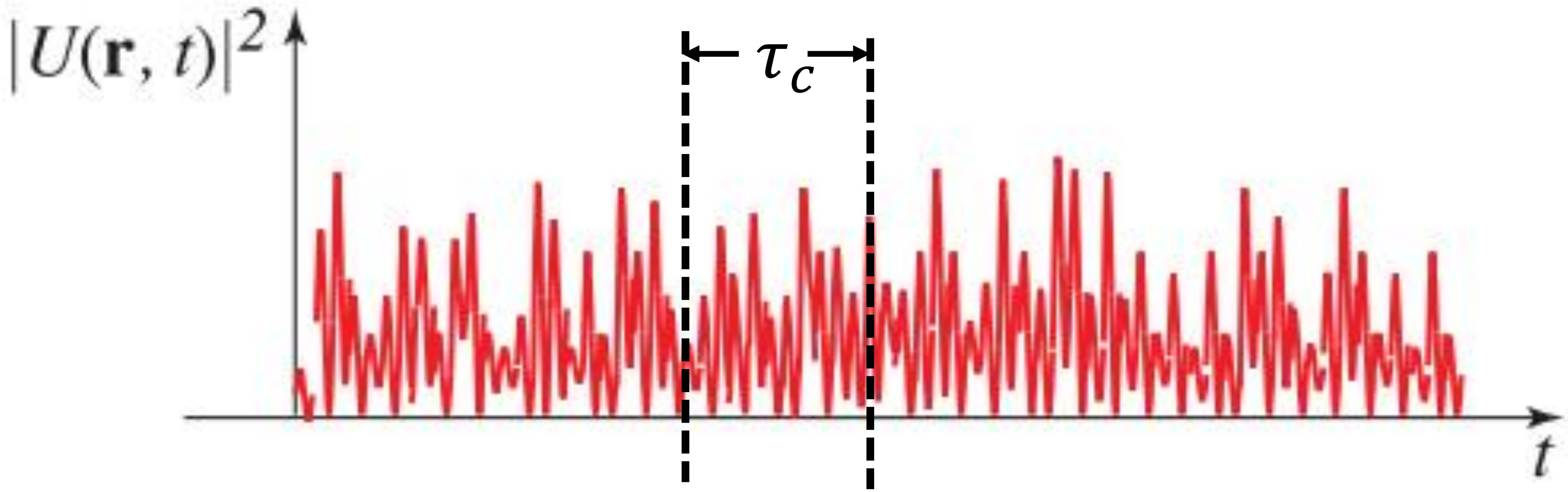
Introduction – 1st Order



τ_c - Longitudinal Coherence Time,
so Longitudinal Coherence Length L_c = **c** τ_c where **c** is the speed of light

Fundamentals of Photonics, Saleh and Teich

Introduction – 1st Order

- To get coherence time we measure the auto-correlation function of the electric field.

$$G(\tau) = \langle E(t)E^*(t+\tau)\rangle$$

- The normalised auto-correlation of the electric field is called the **coherence function**.

$$g^{(1)}(\tau) = \frac{\langle E(t)E^*(t+\tau)\rangle}{\langle E(t)E^*(t)\rangle}$$

- From this temporal coherence function one can quantitatively calculate the coherence time of the field.

Introduction – 1st Order

- The same can be defined along with the spatial separation Δr.

$$g^{(1)}(\Delta r, \tau) = \frac{\langle E(r_1, t) E^*(r_2, t+\tau) \rangle}{\langle E(r_1, t) E^*(r_2, t) \rangle} \qquad \Delta r = |r_1 - r_2|$$

- The temporal delay τ is along the longitudinal direction and the spatial separation Δr is along the transverse direction.

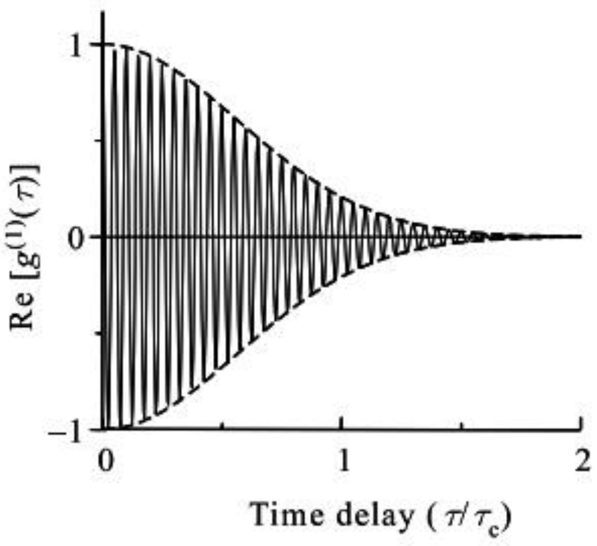
- $g^{(1)}(\Delta r = 0, \tau)$ gives the **temporal coherence function** and $g^{(1)}(\Delta r, \tau = 0)$ gives the **spatial coherence function**.

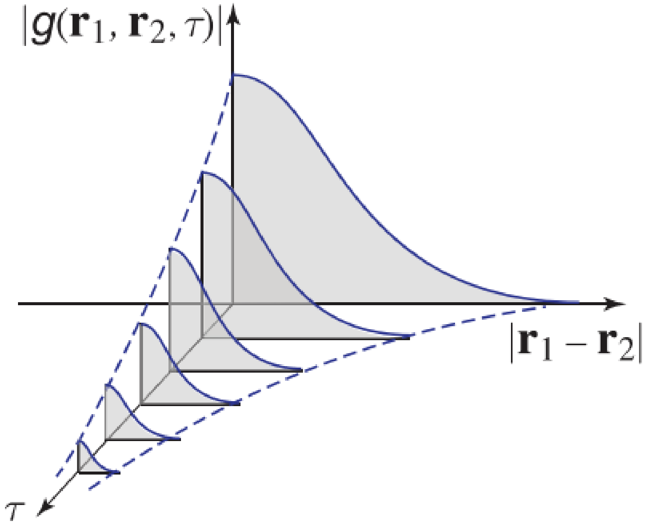
Introduction – 1st Order

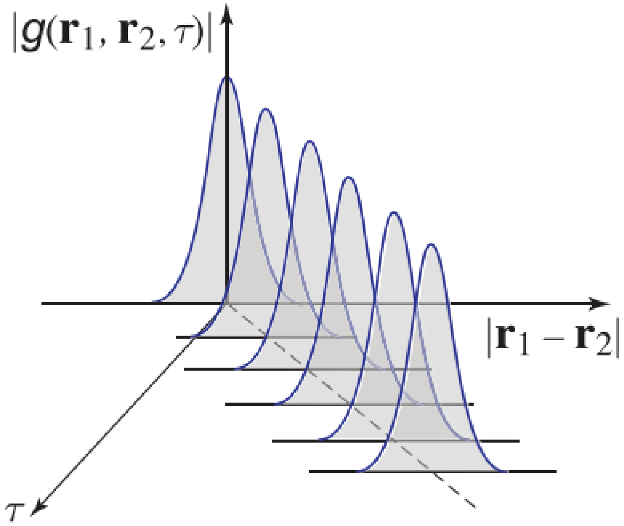
- The same can be defined along with the spatial separation Δr.

$$g^{(1)}(\Delta r, \tau) = \frac{\langle E(r_1, t)E^*(r_2, t+\tau)\rangle}{\langle E(r_1, t)E^*(r_2, t)\rangle} \qquad \Delta r = |r_1 - r_2|$$

- This function is called the **first-order coherence function** and $\left|g^{(1)}(\Delta r, \tau)\right|$ is called **the degree of first-order coherence function**.







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Fundamentals of Photonics, Saleh and Teich

Introduction – 1st Order

- If we consider a quasi monochromatic light with central frequency ω_o which changes with time by $\varphi(t)$,

$$E = E_O e^{-i\omega_o t} e^{i\varphi(t)}$$

- Then $g^{(1)}(\tau) = e^{-i\omega_o t}\left\langle e^{i[\varphi(t+\tau)-\varphi(t)]}\right\rangle$ and so $\boldsymbol{0 \leq |g^{(1)}(\tau)| \leq 1}$

Description of light	Spectral width	Coherence	Coherence time	$\lvert g^{(1)}(\tau)\rvert$
Perfectly monochromatic	0	Perfect	Infinite	1
Chaotic	$\Delta\omega$	Partial	$\sim 1/\Delta\omega$	$1 > \lvert g^{(1)}(\tau)\rvert > 0$
Incoherent	Effectively infinite	None	Effectively zero	0

Introduction – 1st Order

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- Depending on the underlying spectral broadening mechanism the functional form of $g^{(1)}(\tau)$ varies.

- For Natural broadening $\boldsymbol{g^{(1)}(\tau) = e^{-i\omega_o t}\exp\left(\frac{-|\tau|}{\tau_c}\right)}$ where $\tau_c = {}^{1}/_{\Delta\omega}$

  For Doppler broadening $\boldsymbol{g^{(1)}(\tau) = e^{-i\omega_o t}\exp\left(\frac{-\pi}{2}\left(\frac{\tau}{\tau_c}\right)^2\right)}$ where $\tau_c = {}^{8\pi ln2^{1/2}}/_{\Delta\omega}$

Introduction – 1st Order

- According to Weiner Khintchine Theorem, the spectral distribution is the Fourier transform of the first-order temporal coherence function.

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g^{(1)}(\tau)\exp(i\omega\tau)\,d\tau$$

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c_o\tau_c$
Filtered sunlight ($\lambda_o = 0.4$–$0.8\ \mu$m)	3.74×10^{14}	2.67 fs	800 nm
Light-emitting diode ($\lambda_o = 1\ \mu$m, $\Delta\lambda_o = 50$ nm)	1.5×10^{13}	67 fs	20 μm
Low-pressure sodium lamp	5×10^{11}	2 ps	600 μm
Multimode He–Ne laser ($\lambda_o = 633$ nm)	1.5×10^{9}	0.67 ns	20 cm
Single-mode He–Ne laser ($\lambda_o = 633$ nm)	1×10^{6}	1 μs	300 m

Measurement – 1st Order

- The degree of coherence can be measured from the **interference** experiments.

- If we interfere light beams with $E(t)$ and $E(t+\tau)$ then,

$$I = \langle |E(t)|^2 \rangle + \langle |E(t+\tau)|^2 \rangle + \langle E(t)E^*(t+\tau) \rangle + \langle E^*(t)E(t+\tau) \rangle$$

$$I = I_1 + I_2 + 2Re(\langle E(t)E^*(t+\tau) \rangle)$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, Re\left(g^{(1)}(\tau)\right)$$

$$\boldsymbol{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\left|g^{(1)}(\tau)\right| cos\varphi}$$

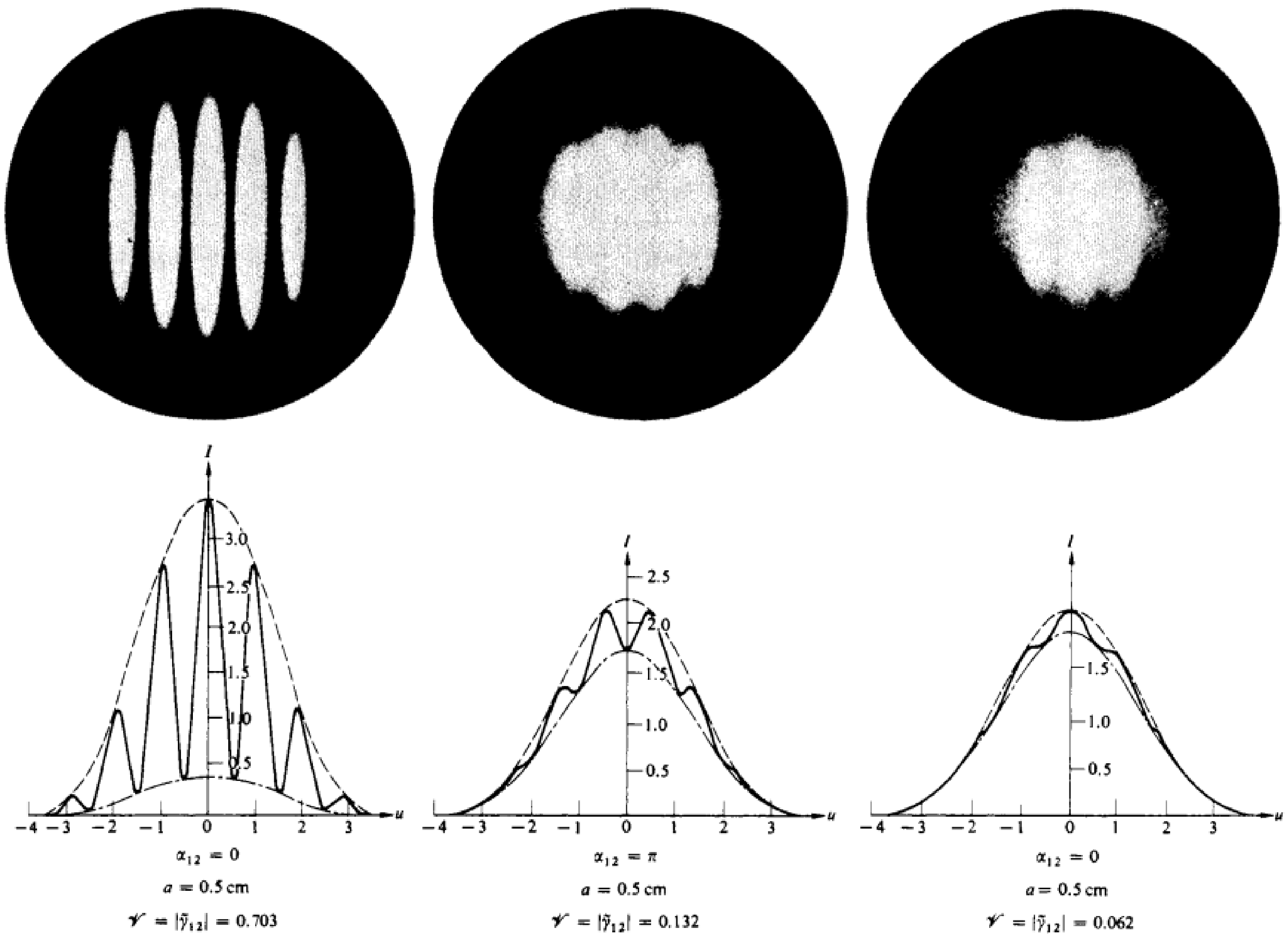
- This has the form $I = I_1 + I_2 + 2\gamma\, cos\varphi$ where γ is called **visibility.**

Measurement – 1st Order

$$\text{Visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} \left| g^{(1)}(\tau) \right|$$

From measuring the visibility of the fringe one can calculate the degree of first order coherence.

Measurement – 1st Order



Optics, Eugene Hecht and A.R.Ganesan

Measurement – 1st Order

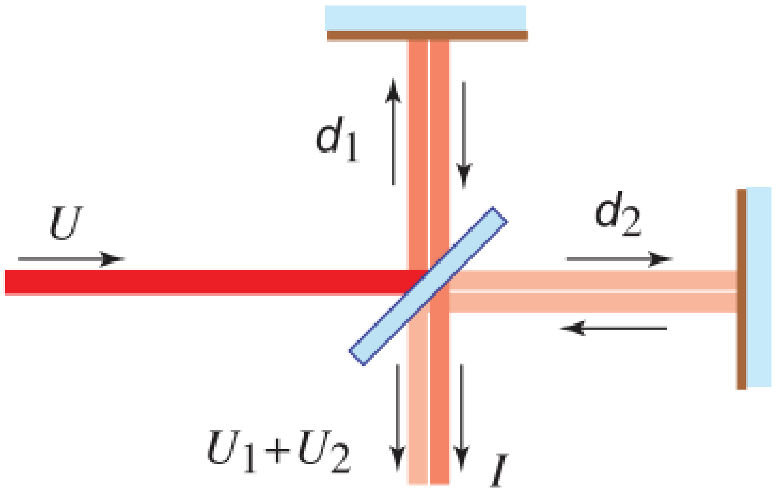
$$\text{Visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}\left|g^{(1)}(\tau)\right|$$

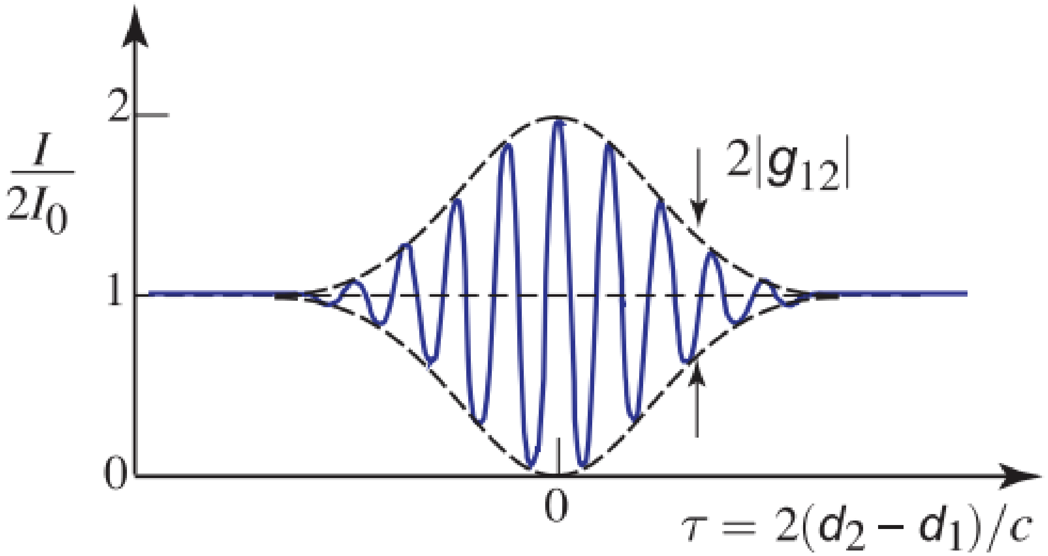
From measuring the visibility of the fringe one can calculate the degree of first order coherence.

Now what interferometer should we use?

Measurement – 1st Order

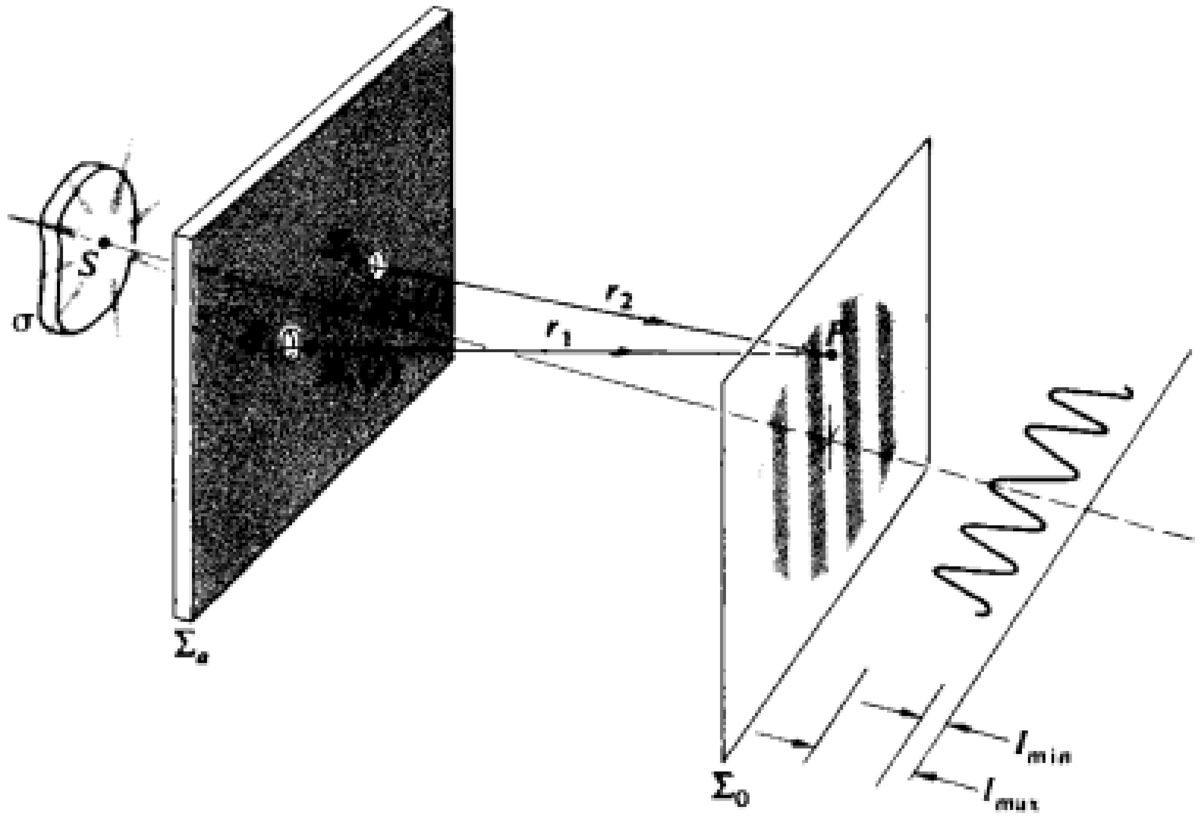
- For measuring temporal coherence we use **Michelson Interferometer**





Measurement – 1st Order

- For measuring spatial coherence we use **Young's double slit experiment**



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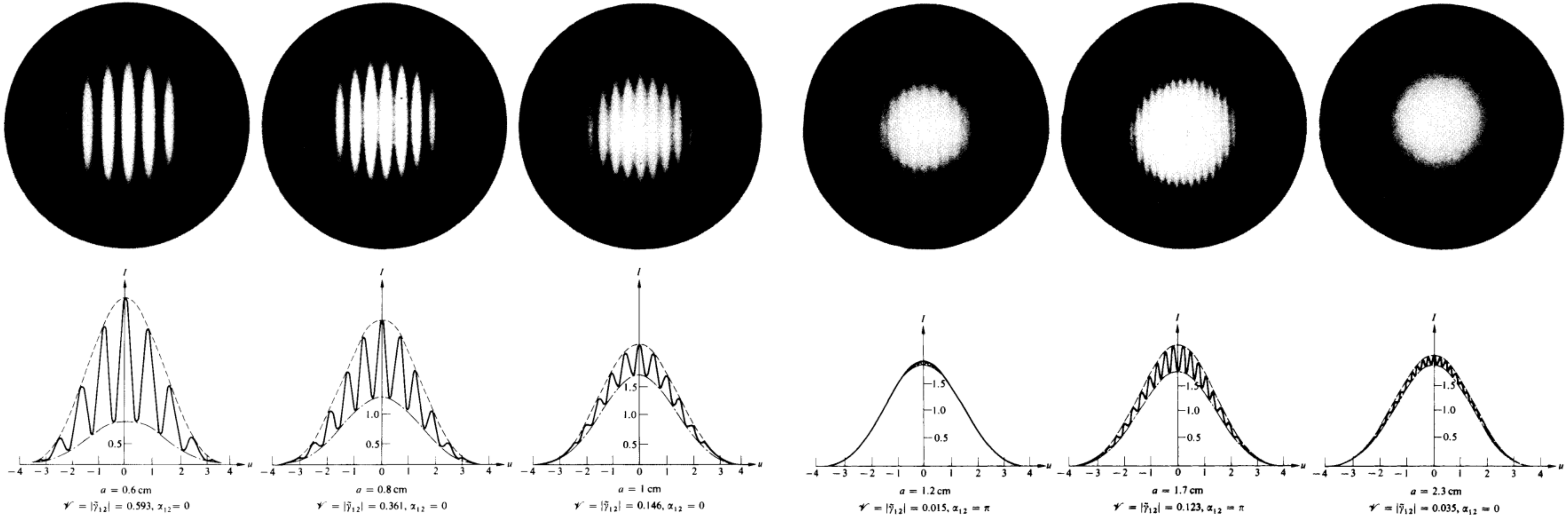
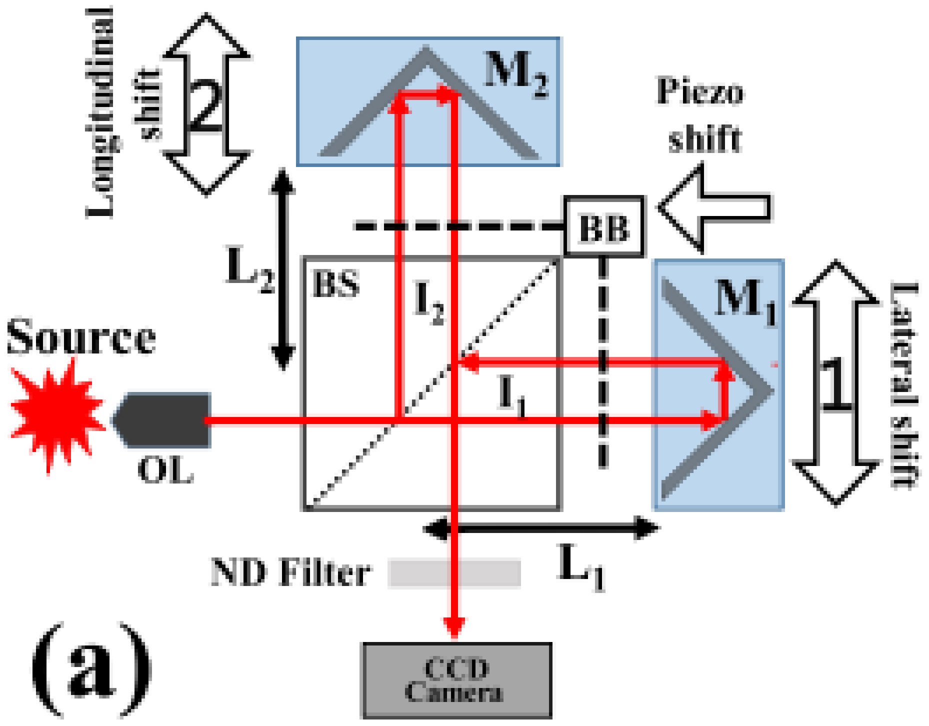
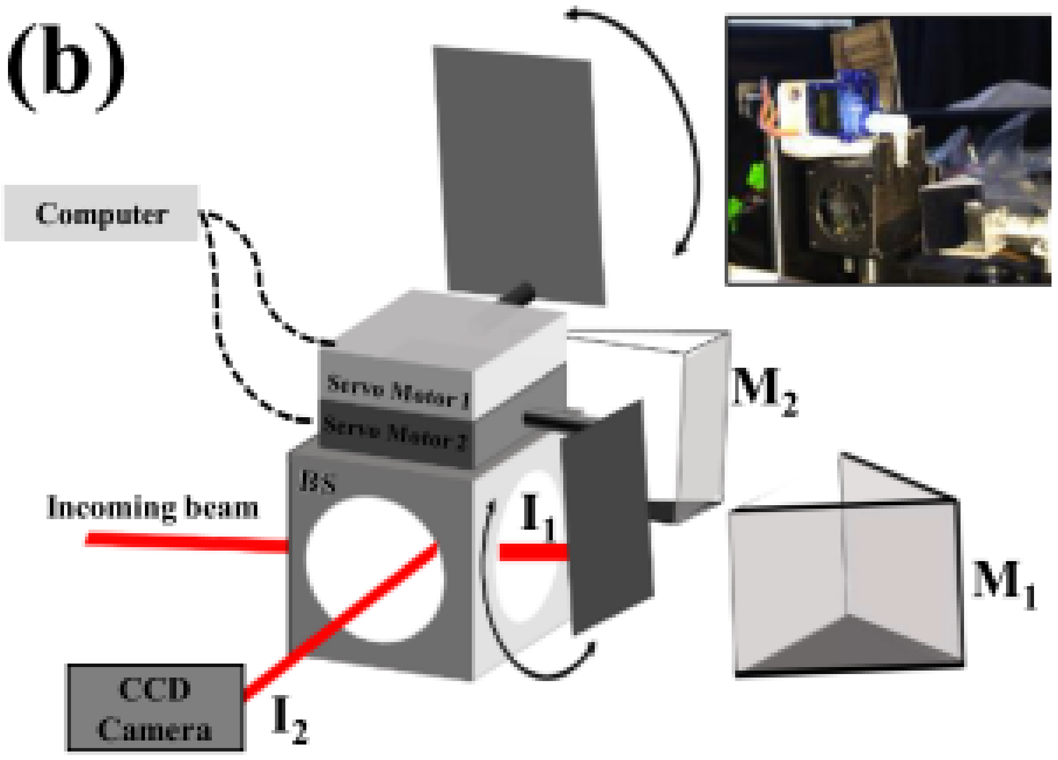
Measurement – 1st Order



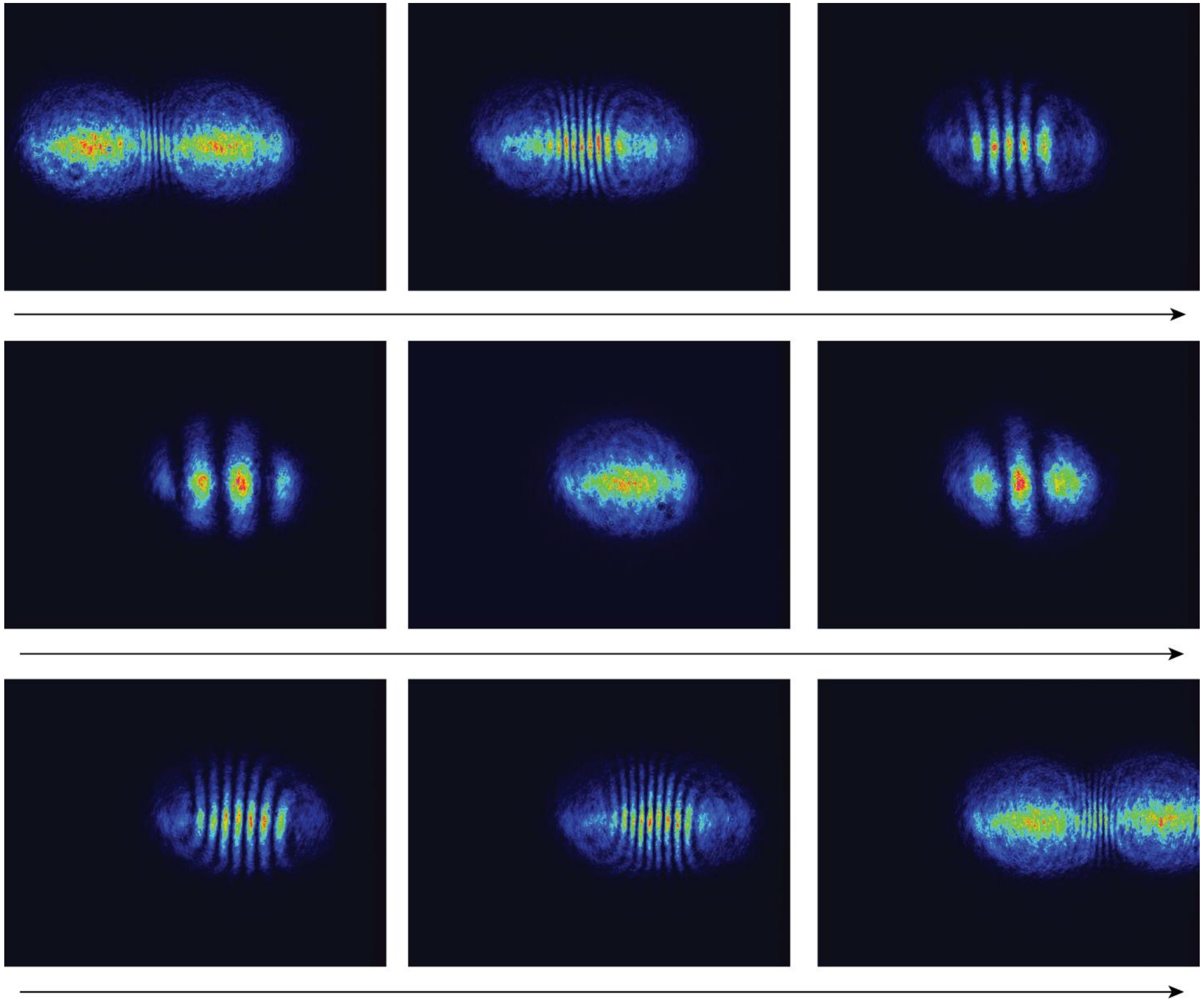
Measurement – 1st Order





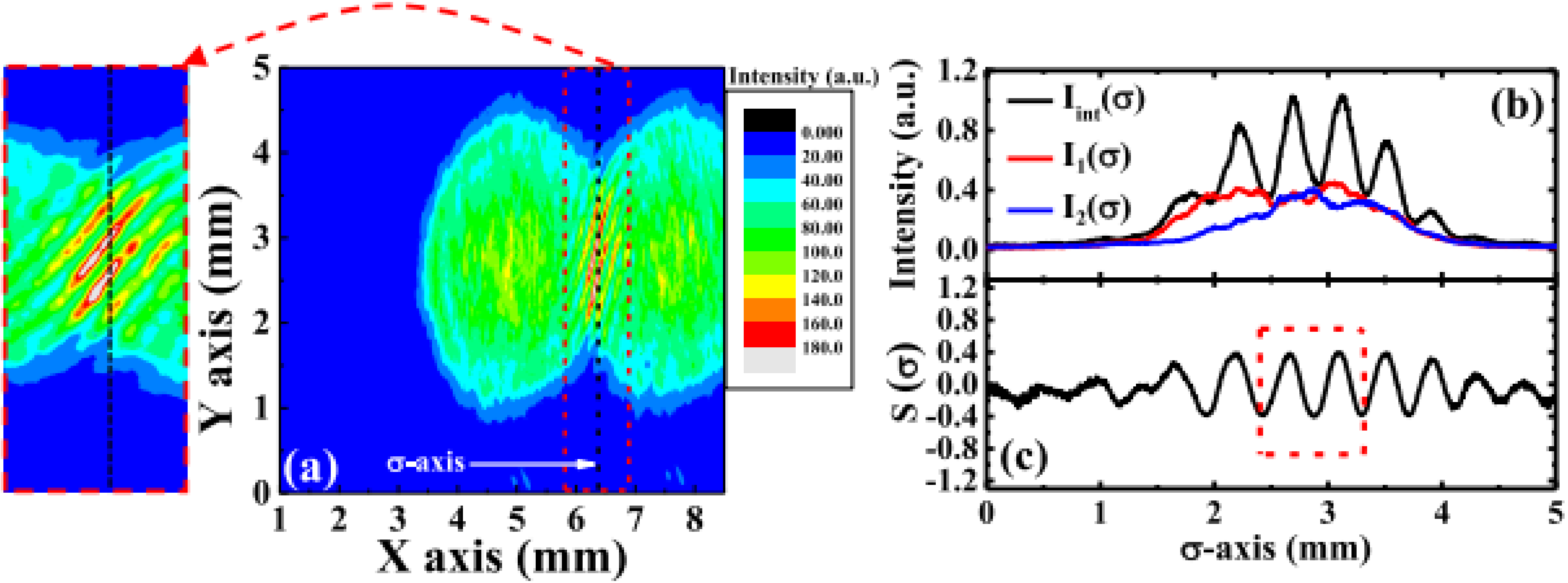
Rev. Sci. Instrum. 92, 105109 (2021)

Measurement – 1^{st} Order



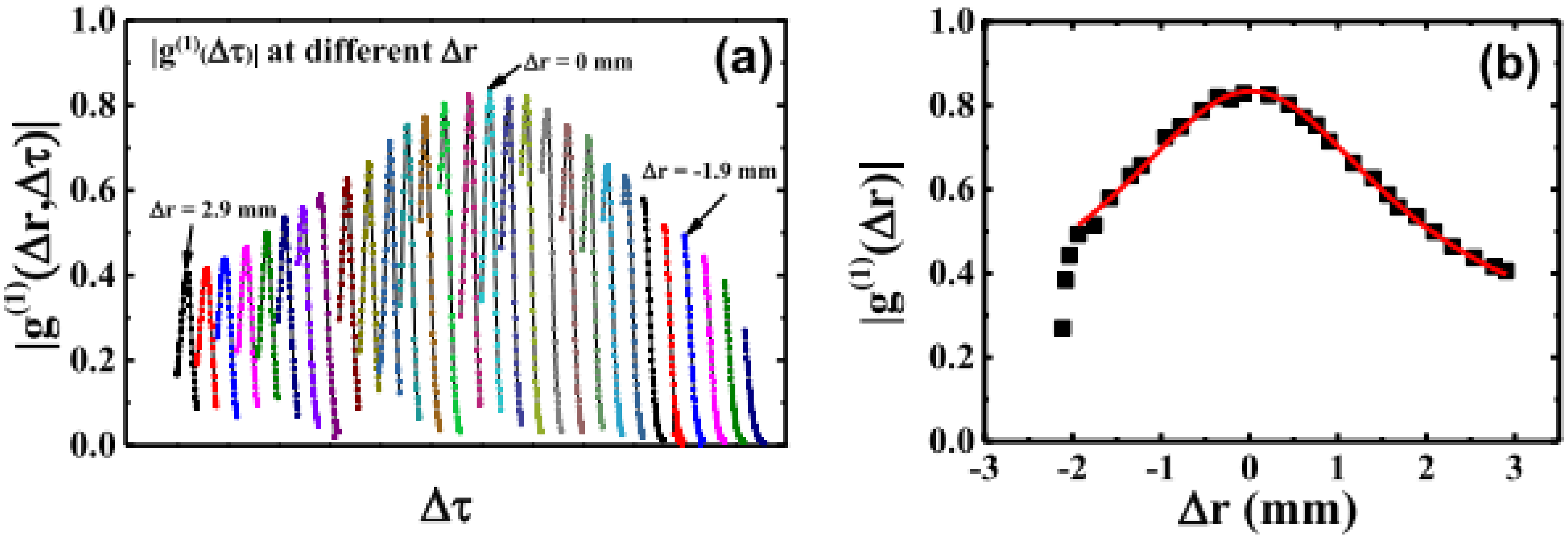
Data from Dr. Shouvik Datta Lab

Measurement – 1st Order



Measurement – 1st Order

- Temporal filtering

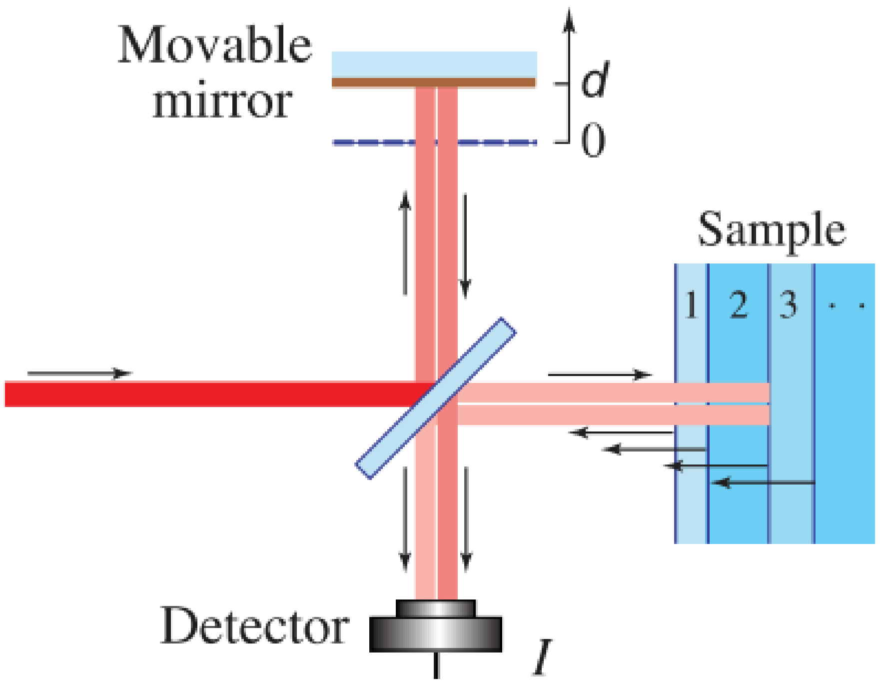


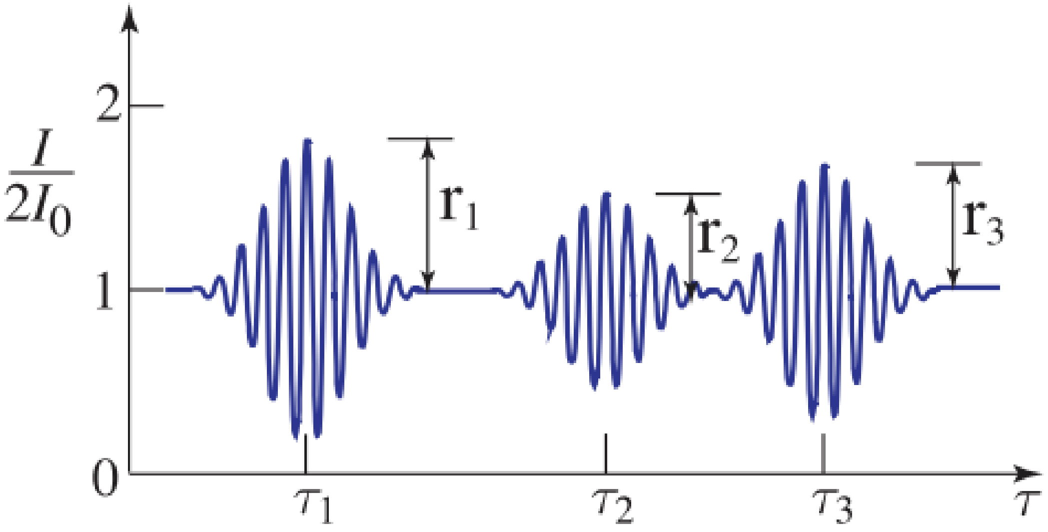
Measurement – 1st Order

Let us see some places where these measurments are used

Application – 1st Order

- Use in **Optical Coherence Tomography (OCT)**





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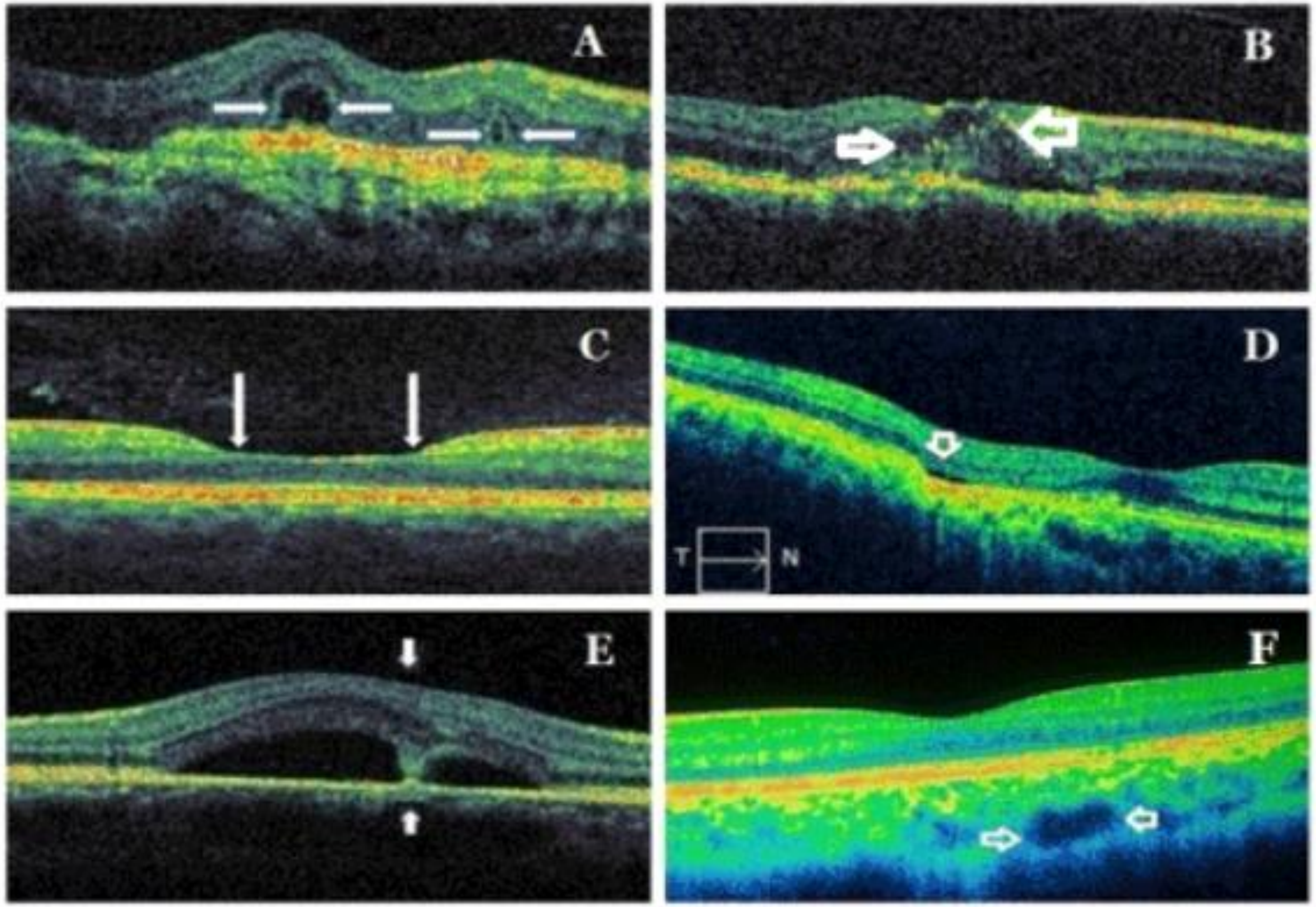
Application – 1st Order

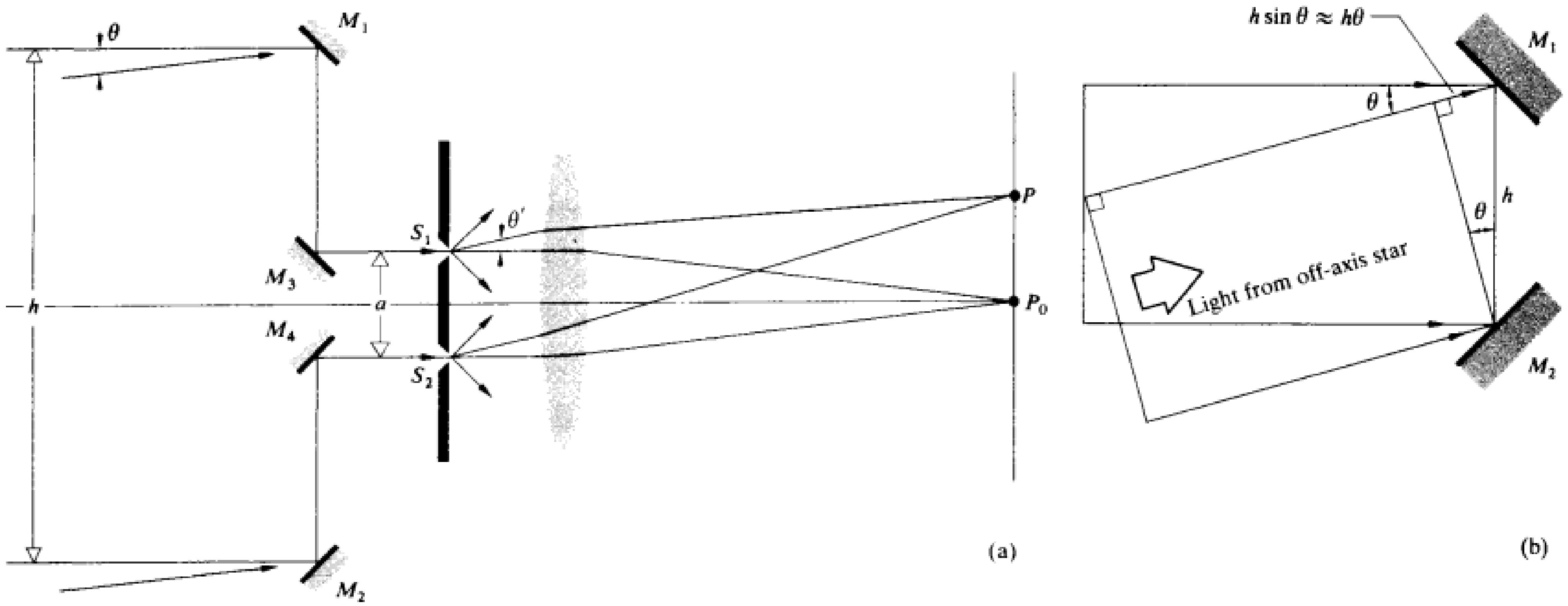
- OCT has main application in **Ophthalmology** 10.15761/NFO.1000130



Turgut B, Demir T (2016) The new landmarks, findings and signs in optical coherence tomography. New Front Ophthalmol 2

Application – 1st Order

- It also used in astronomy to measure angular radius of the planets by **Michelson stellar interferometry**



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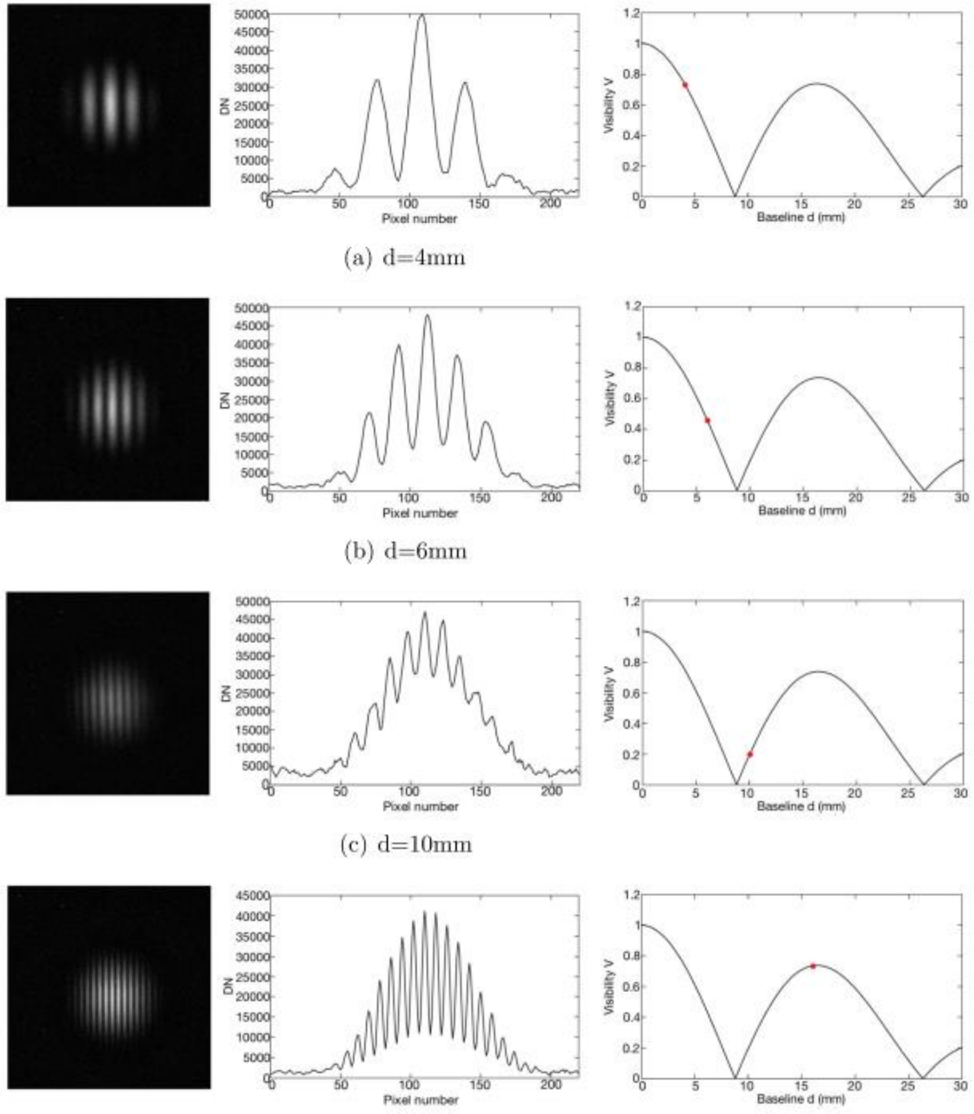
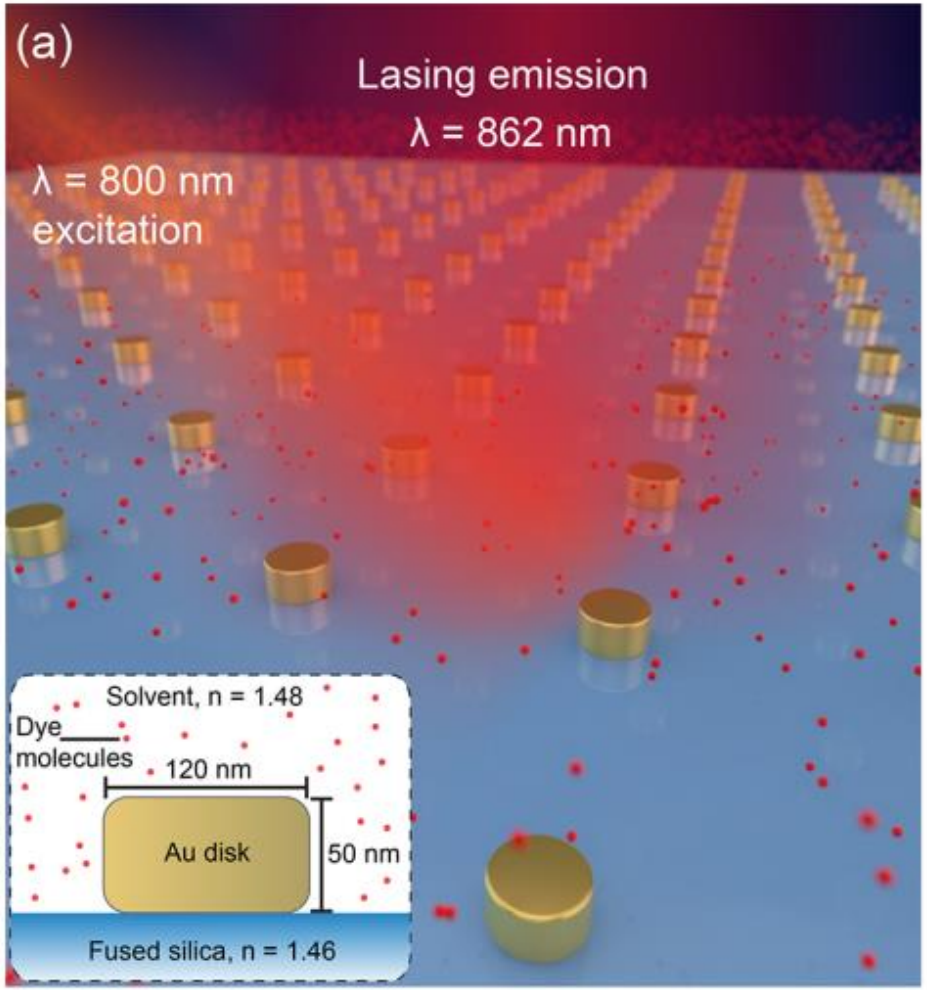


Eur. J. Phys. 38 (2017) 045704

Application – 1st Order

- For understanding the process of **lasing** in different systems

Plasmonic systems



*Thang B. Hoang, Gleb M. Akselrod, Ankun Yang, Teri W. Odom, and Maiken H. Mikkelsen Nano Letters **2017** 17 (11), 6690-6695*

Application – 1st Order

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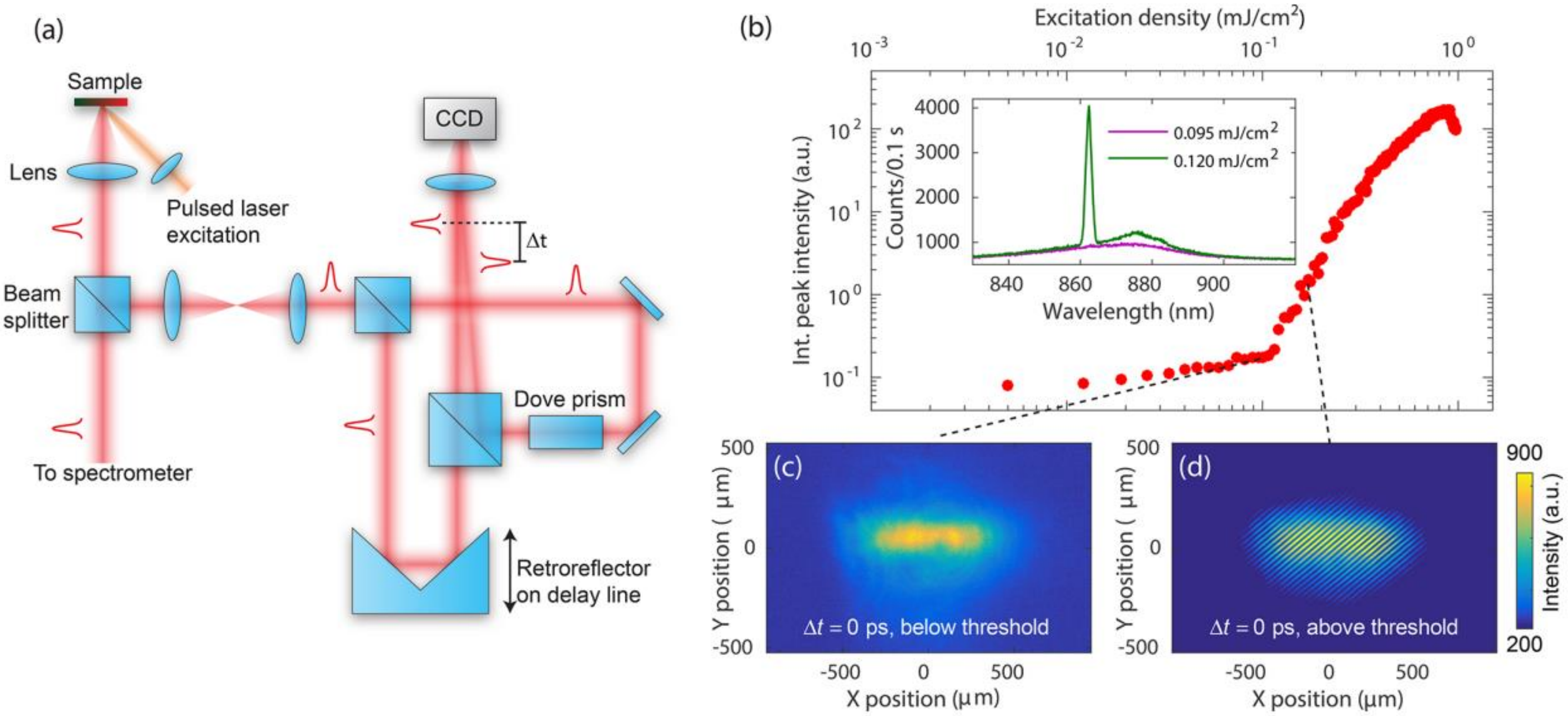

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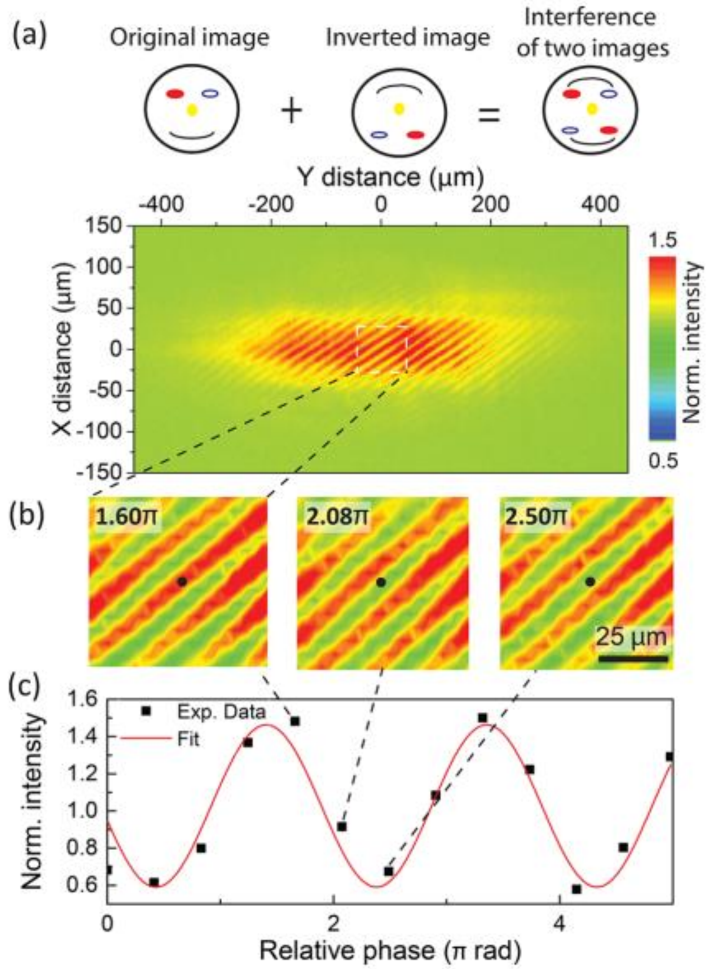

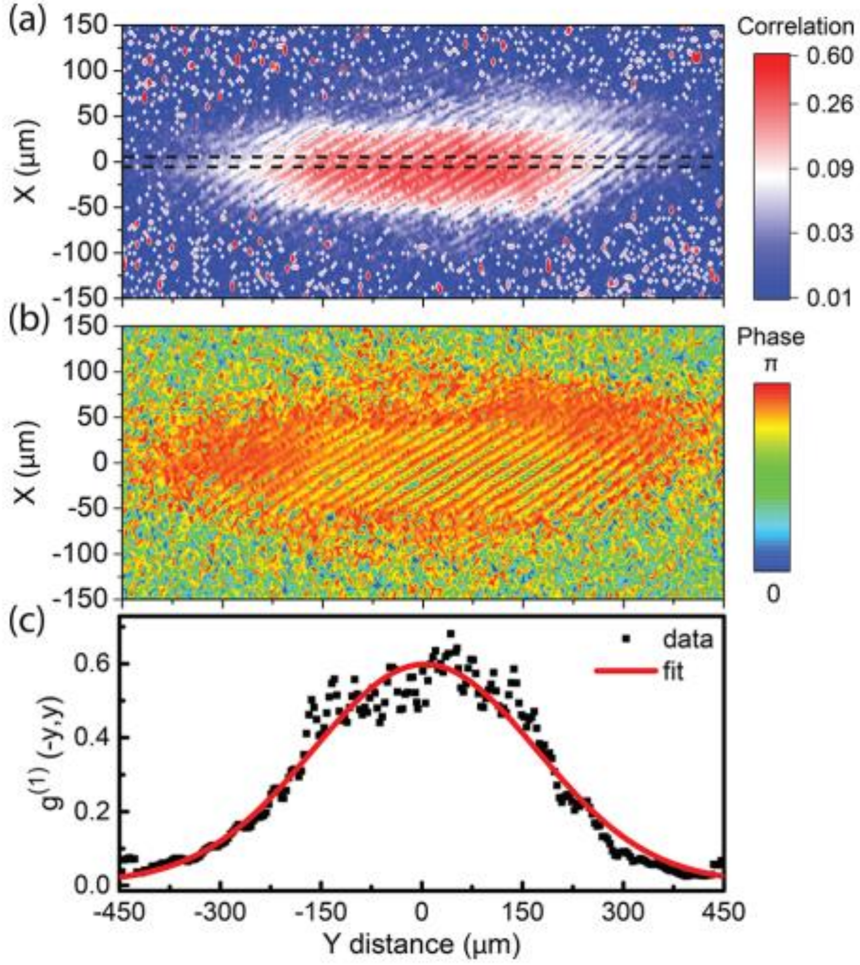

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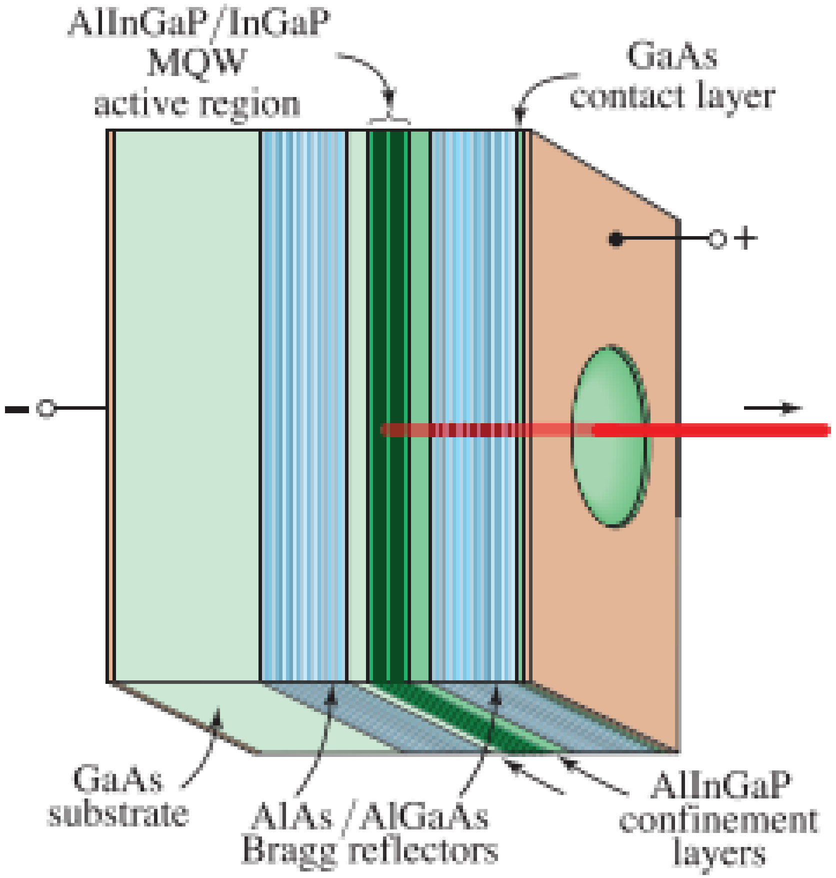
Application – 1st Order

- For understanding the process of **lasing** in different systems

Laser diodes



Fundamentals of Photonics, Saleh and Teich

Application – 1st Order

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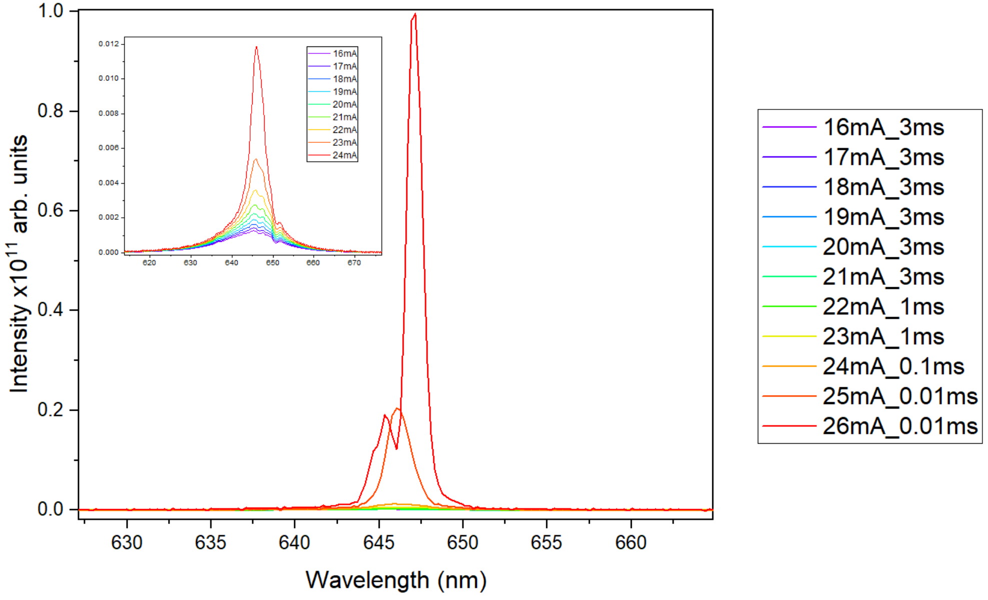

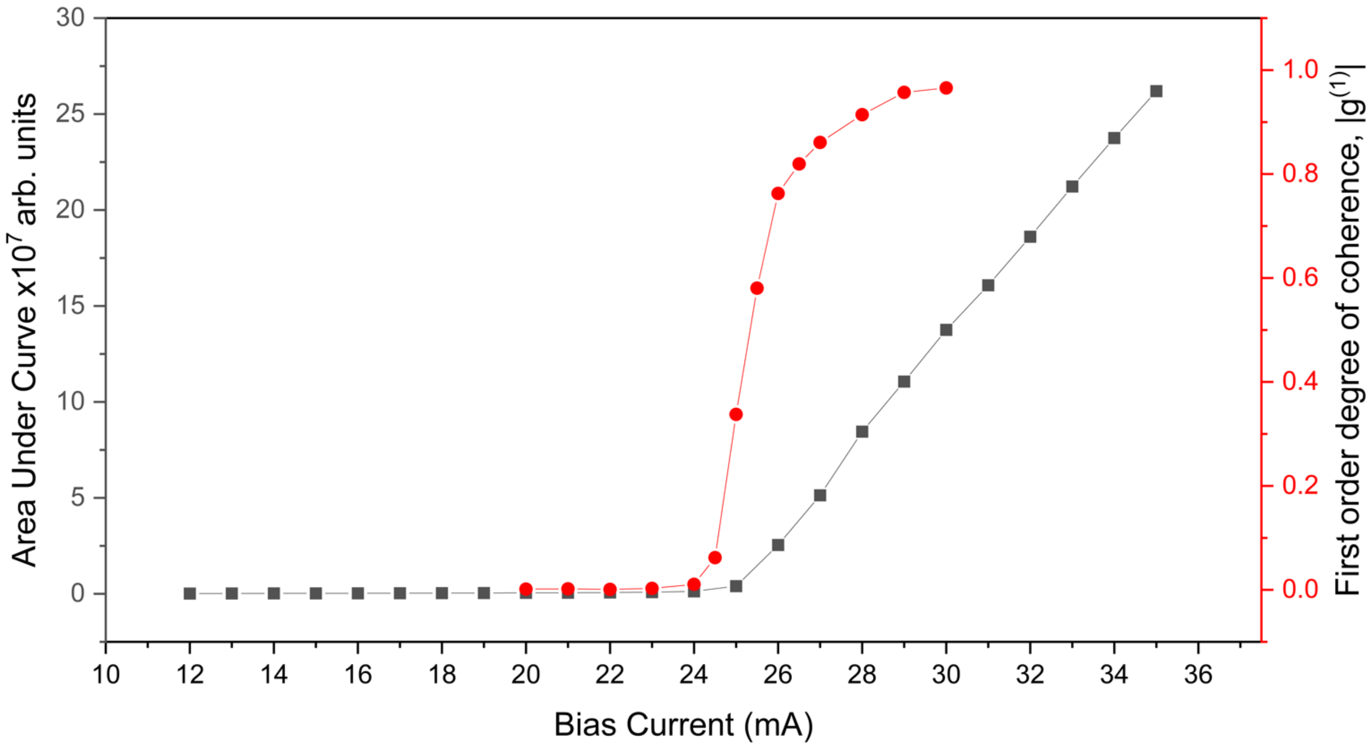

Data from Dr. Shouvik Datta Lab

Application – 1st Order

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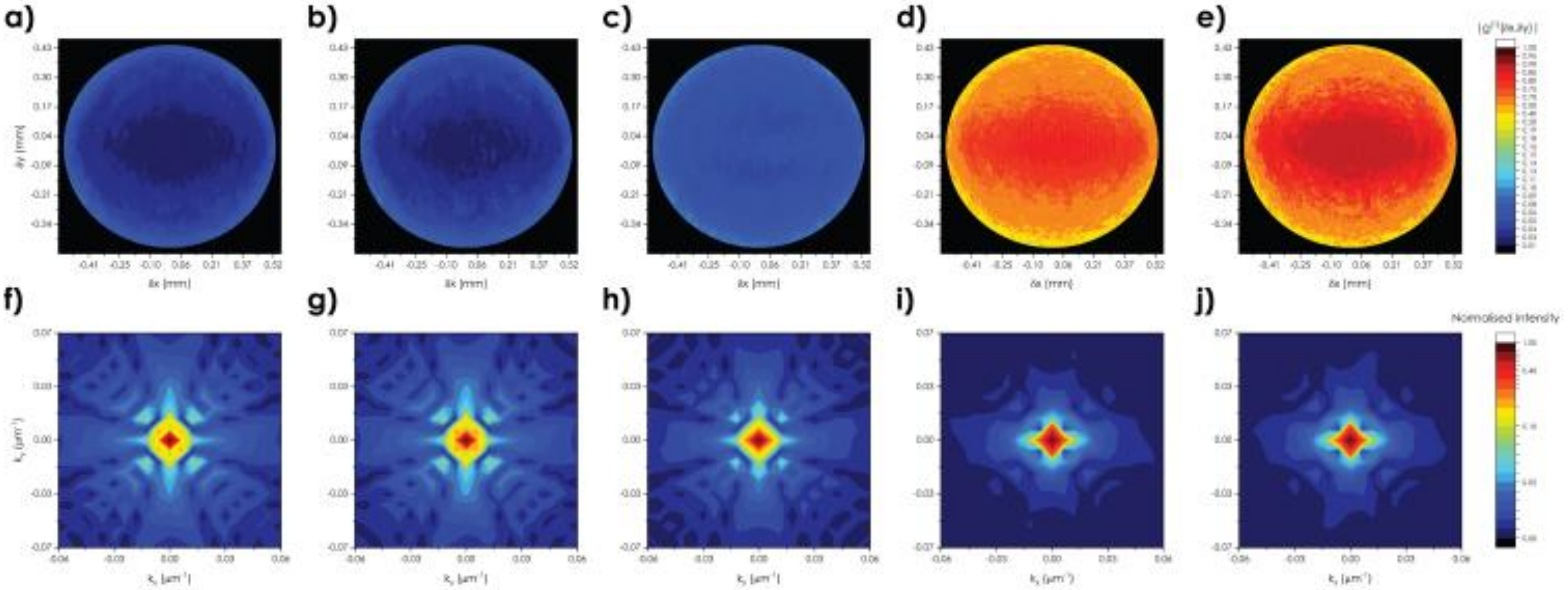

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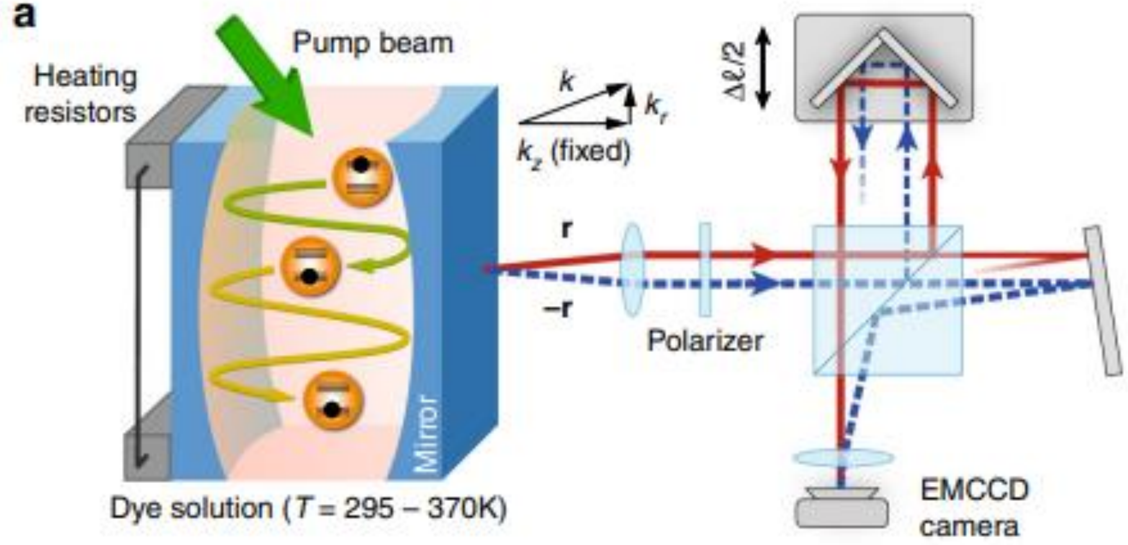
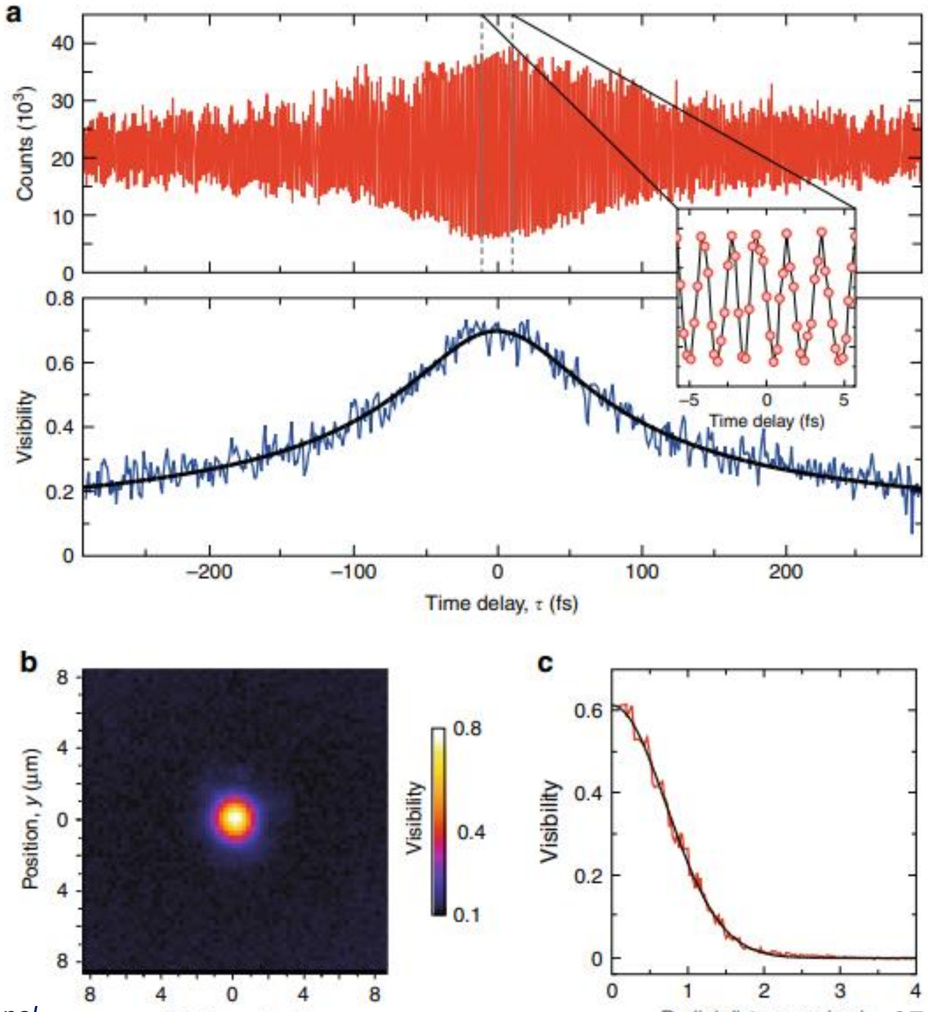
Application – 1st Order

- For studying the **Bose Einstein Condensates (BEC)**

Photonic condensates see when condensate forms





Damm, T., Dung, D., Vewinger, F. et al. First-order spatial coherence measurements in a thermalized two-dimensional photonic quantum gas. Nat Commun 8, 158 (2017)

Application – 1st Order

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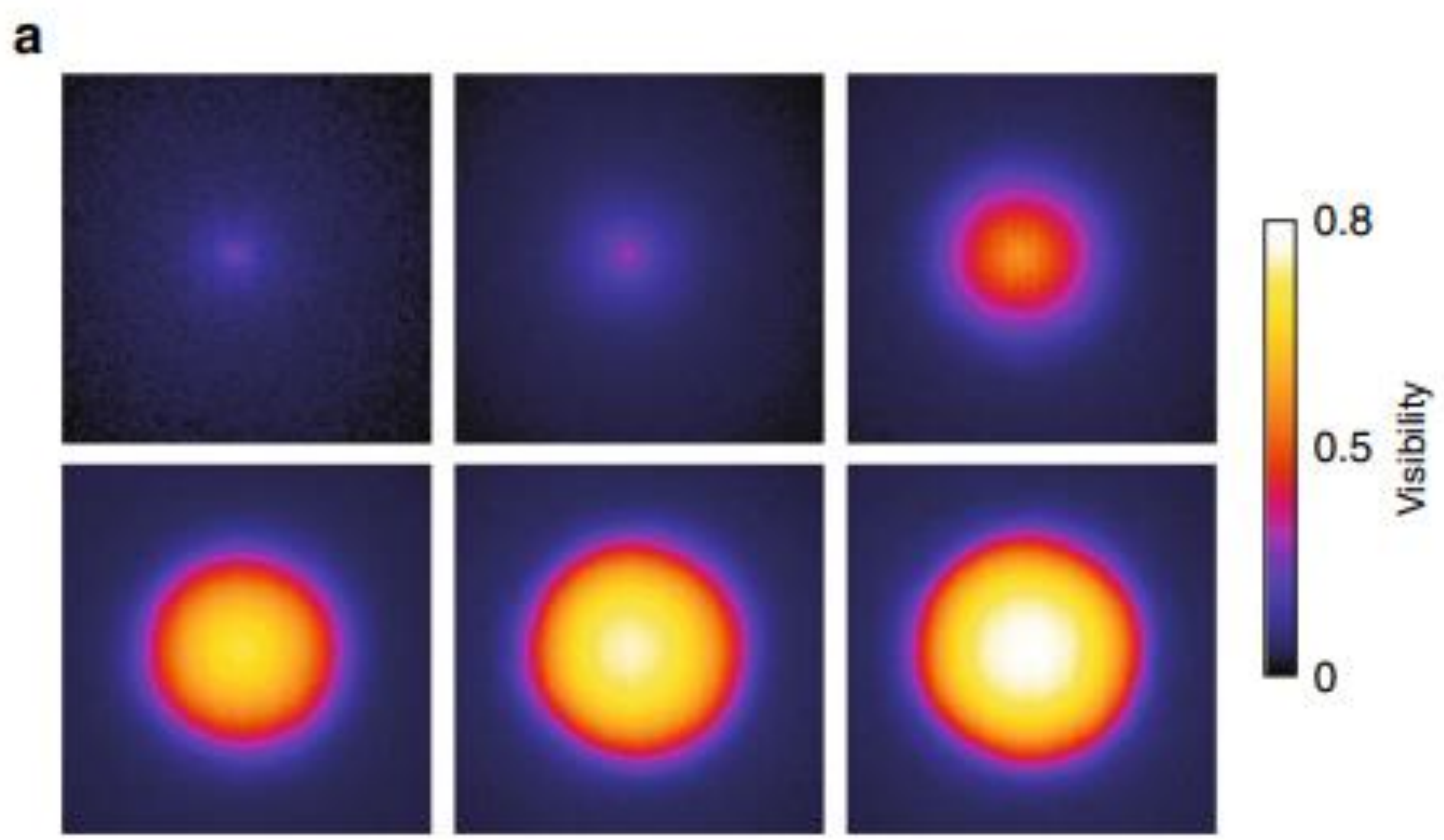

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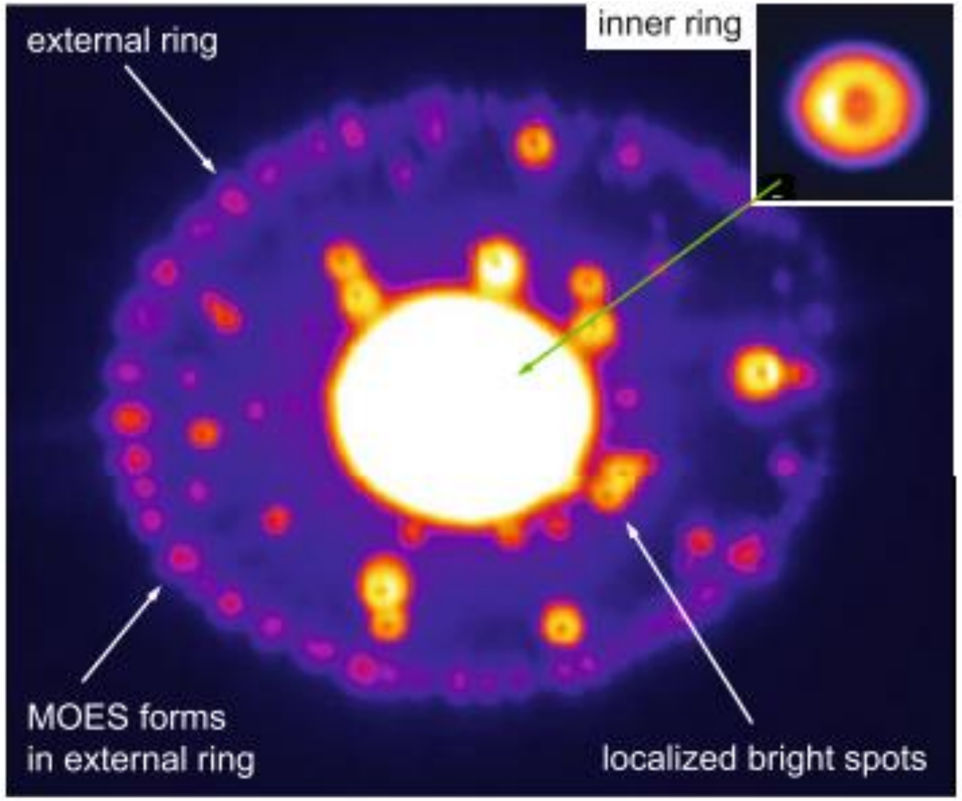
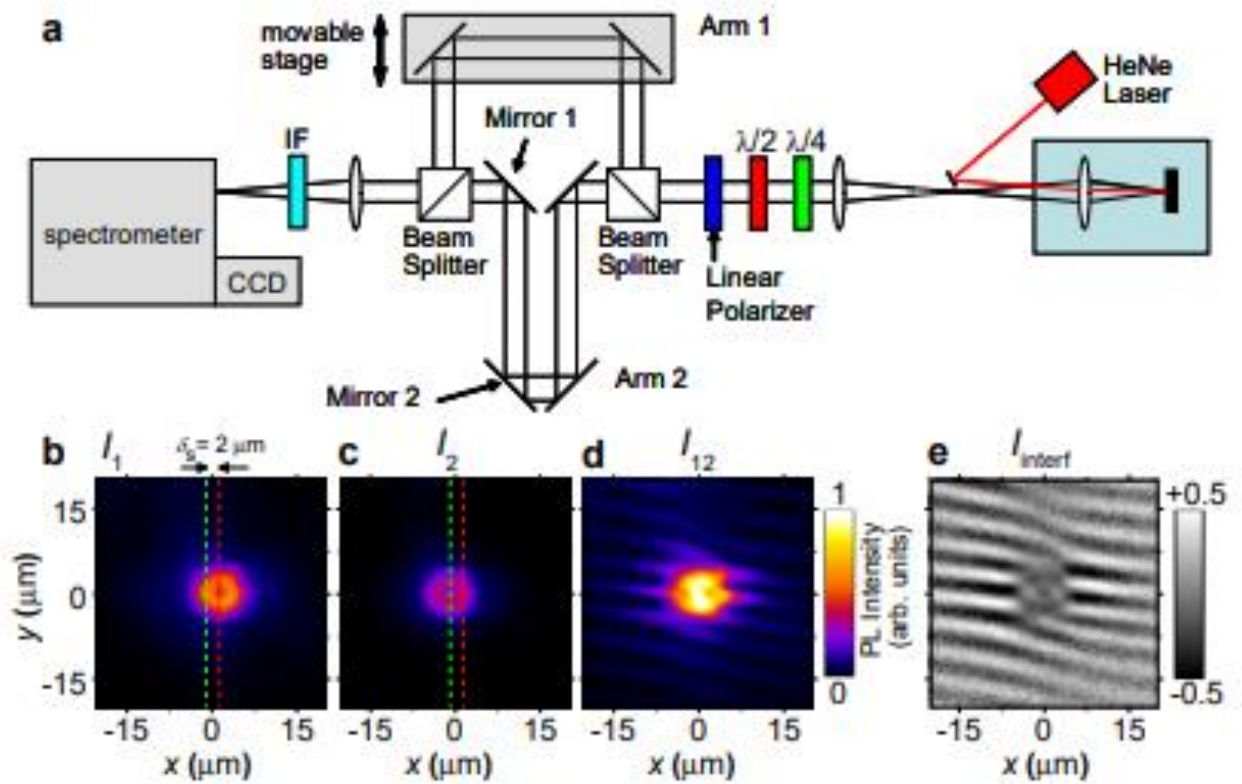
Application – 1st Order

- For studying the **Bose Einstein Condensates (BEC)**

Condensates of Indirect Excitons





High, A., Leonard, J., Hammack, A. et al. Spontaneous coherence in a cold exciton gas. Nature 483, 584–588 (2012)

Application – 1st Order

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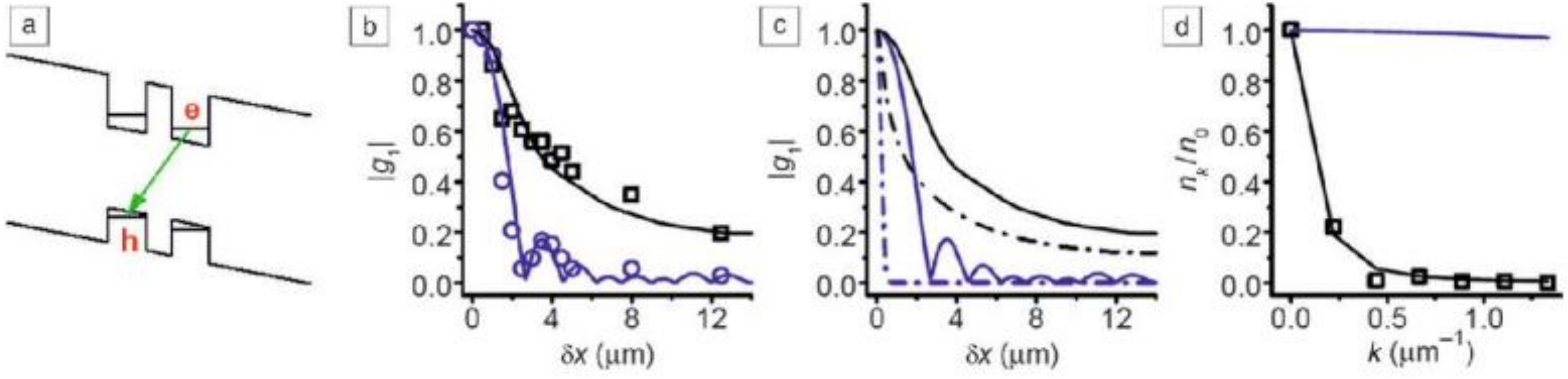
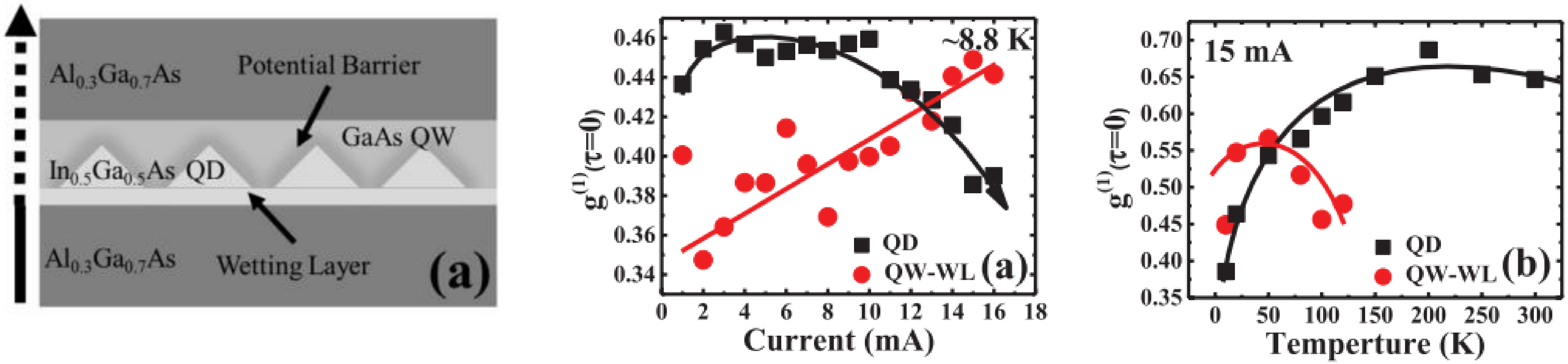


High, A., Leonard, J., Hammack, A. et al. Spontaneous coherence in a cold exciton gas. Nature 483, 584–588 (2012)

Application – 1st Order

- For providing some indirect evidences for processes in **excitonic systems**



Mohit Kumar Singh et al 2019 J. Phys. D: Appl. Phys. 52 095102

Can we have correlations in Intensity?

2^{nd} Order

- So we do second order coherence measurements, **the second order coherence function** is given as

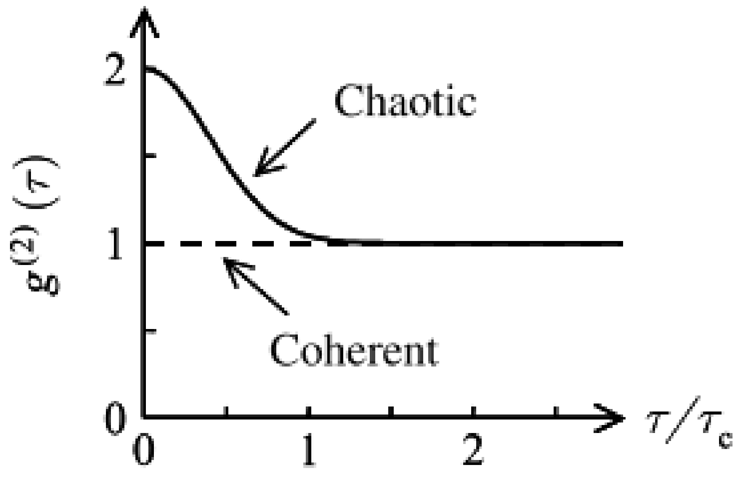
$$g^{(2)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\, E^*(t+\tau)E(t)\rangle}{\langle E(t)E^*(t)\rangle\langle E(t+\tau)E^*(t+\tau)\rangle} = \frac{\langle I(t)I\,(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}$$

$$g^{(2)}(\tau) = \frac{\langle n(t)n'(t+\tau)\rangle}{\langle n(t)\rangle\langle n'(t+\tau)\rangle}$$

2^{nd} Order

- We will discuss the photon picture.
- The **photon flux**, $\varphi = \frac{IA}{\hbar\omega}$ and the **average count rate** $R = \eta \frac{IA}{\hbar\omega}$ where η is the quantum efficiency
- There is a **dead time of 1 μs (approx.) for photodetectors** which limits the photon count measurements.
- For intensity correlation measurements we need low intensity light i.e., photon picture

2nd Order

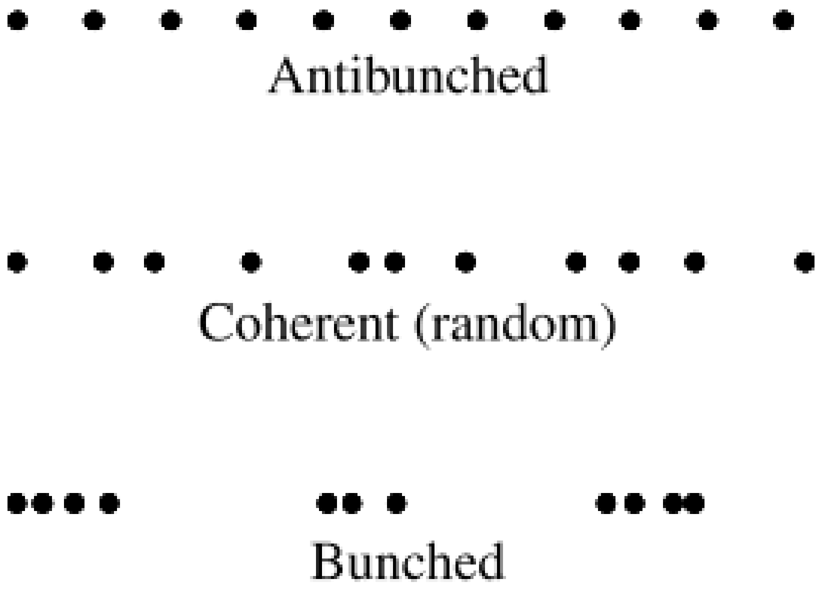


Light source	Property	Comment
All classical light	$g^{(2)}(0) \geq 1$	$g^{(2)}(0) = 1$ when $I(t) = \text{constant}$
	$g^{(2)}(0) \geq g^{(2)}(\tau)$	
Perfectly coherent light	$g^{(2)}(\tau) = 1$	Applies for all τ
Gaussian chaotic light	$g^{(2)}(\tau) = 1 + \exp\left[-\pi(\tau/\tau_c)^2\right]$	τ_c = coherence time
Lorentzian chaotic light	$g^{(2)}(\tau) = 1 + \exp(-2\lvert\tau\rvert/\tau_0)$	τ_0 = lifetime

Quantum Optics – An Introduction, Mark Fox

2^{nd} Order

Classical description	Photon stream	$g^{(2)}(0)$
Chaotic	Bunched	>1
Coherent	Random	1
None	Antibunched	<1



2nd Order

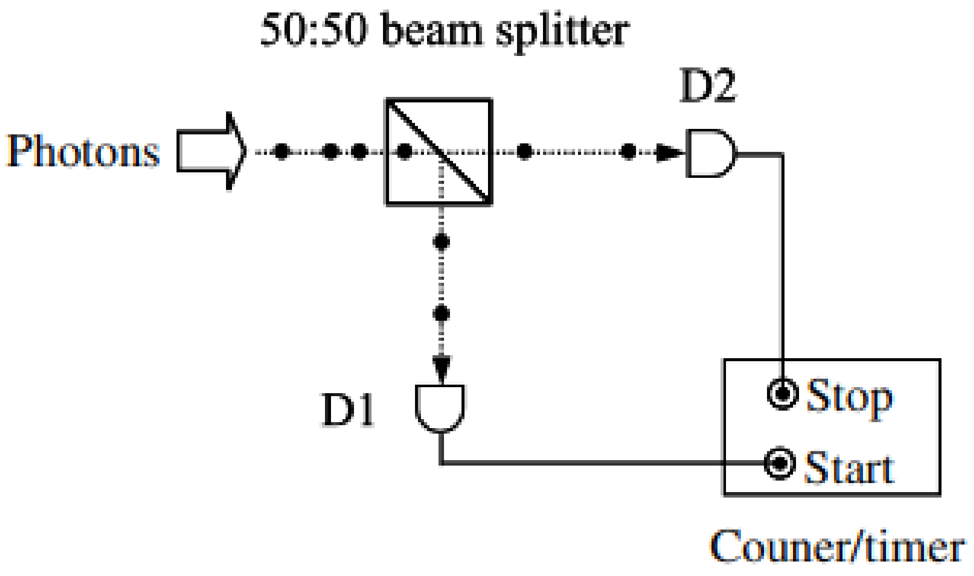
- Now if we calculate the **probability P(n) of finding n photons in within a beam of length L containing n segments** then we get,

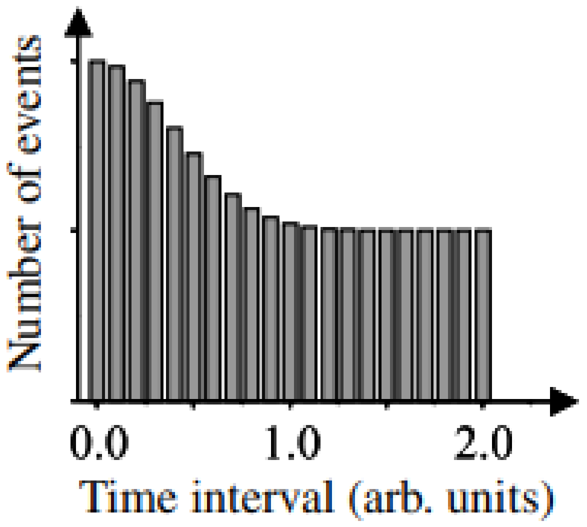
Photon statistics	Classical equivalents	$I(t)$	Δn
Super-Poissonian	Partially coherent (chaotic), incoherent, or thermal light	Time-varying	$> \sqrt{\overline{n}}$
Poissonian	Perfectly coherent light	Constant	$\sqrt{\overline{n}}$
Sub-Poissonian	None (non-classical)	Constant	$< \sqrt{\overline{n}}$

- But measuring statistics directly gives Poissonian mostly

2nd Order

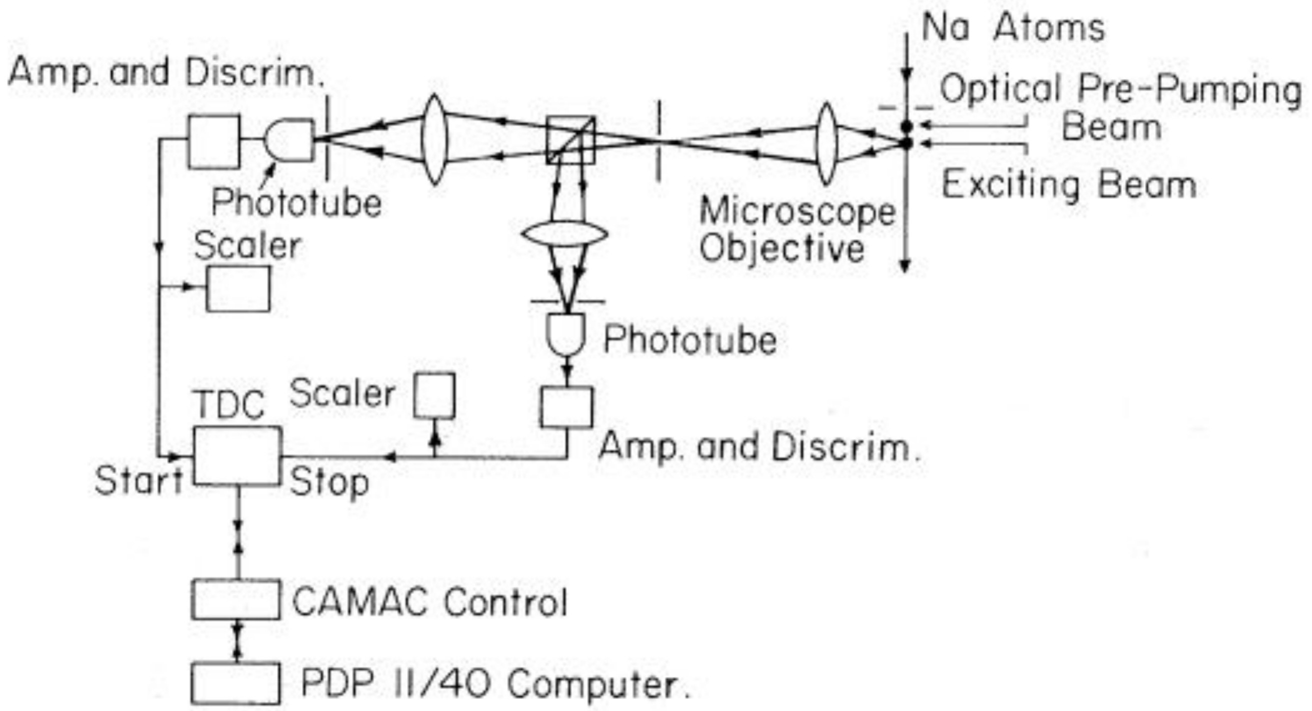
- To measure second order coherence function, **Hanbury Brown Twiss Interferometer** is used

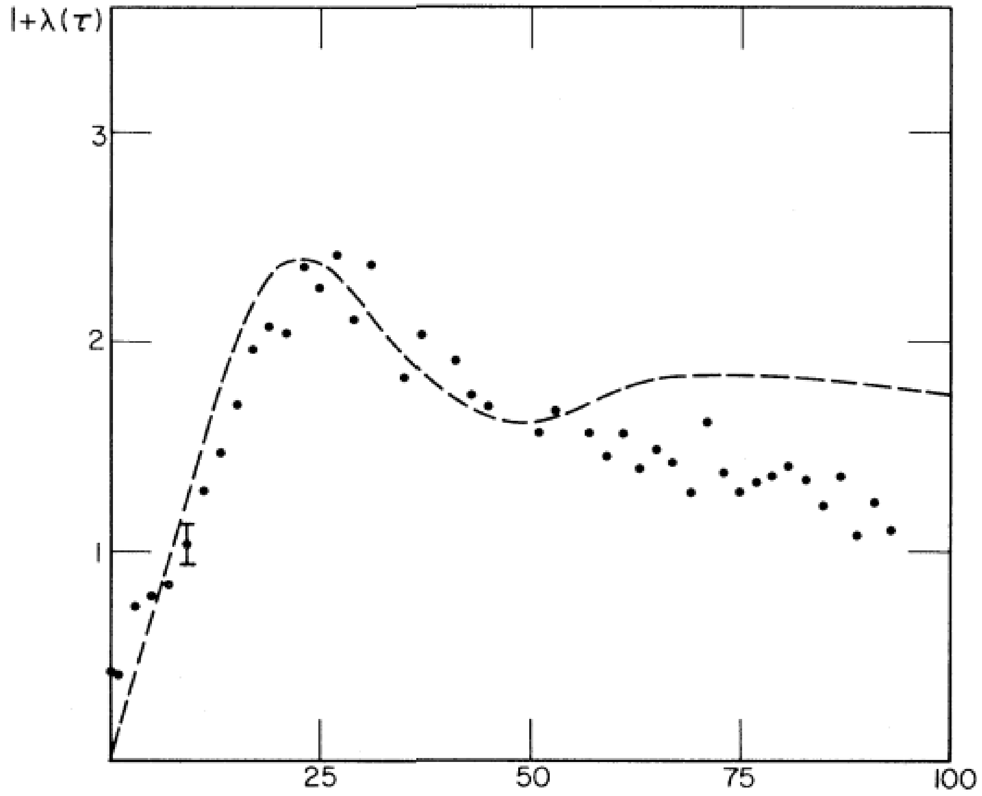




2nd Order

- It can be used to identify the antibunching mechanisms





H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett., 39, 691 (1977)

2nd Order

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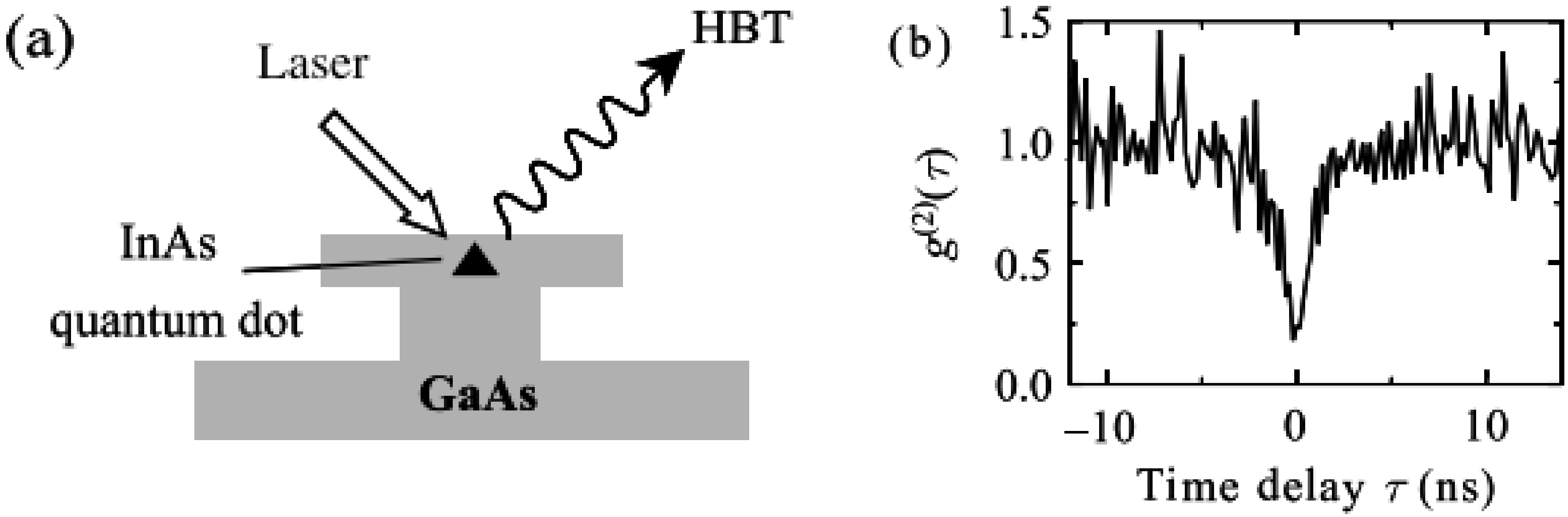
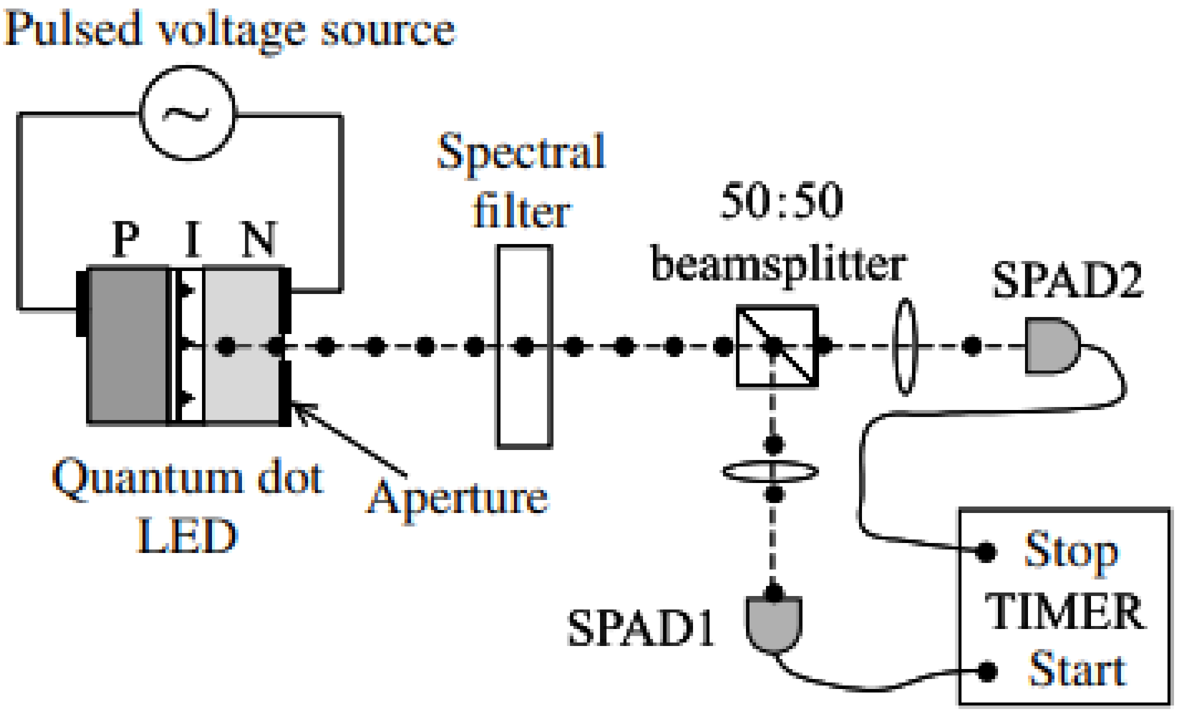
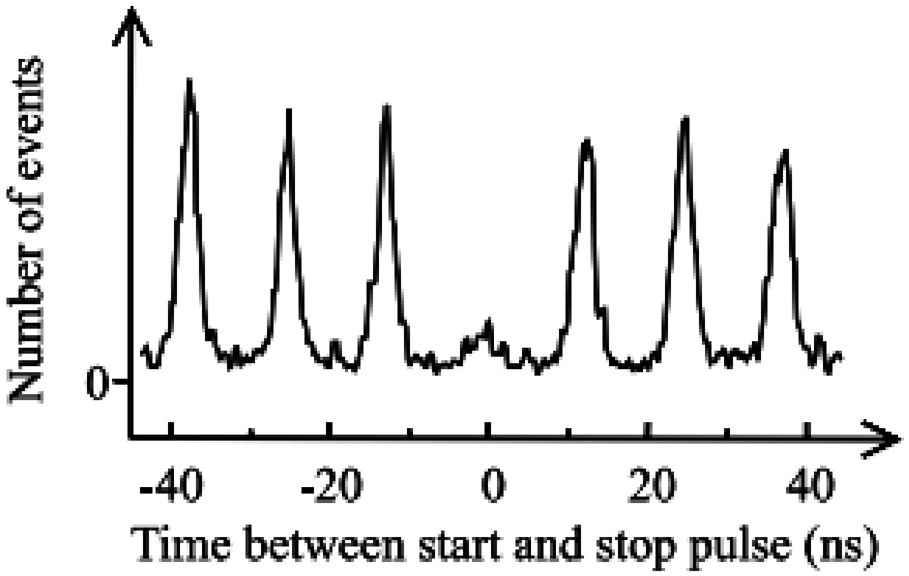


2nd Order

- Realisation of single photon emitters





Conclusions

- The first-order and second-order coherence functions give information about the underlying statistics in systems

- Temporal coherence function is measured in Michelson Interferometer
- Spatial coherence function is measured in Young's double slit interferometer

- **More temporal coherence, lesser spectral width**

  More spatial coherence, more directionality or narrowing in momentum space

- Intensity correlations are measured through HBT interferometer.

- Antibunching behaviour can be used to realise single photon sources