Solar Convection - An Overview

Soumil Kelkar

Surface of the Sun

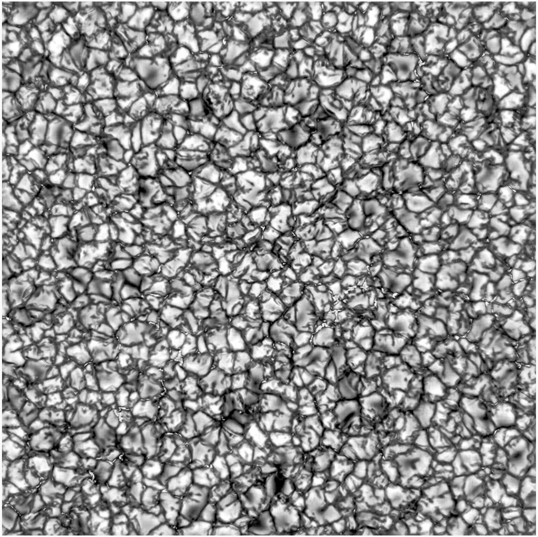

Figure 1

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- Stars are powered by nuclear fusion which converts lighter elements like H into heavier elements like He and so on.
- Energy released in the core has to be transported to the outer layers.
- Three main processes of energy transport:-
  1. Conduction
  2. Radiation
  3. Convection

Convection in the Earth's atmosphere

- Convection involves bulk transport of gas molecules usually driven by thermal fluctuations.

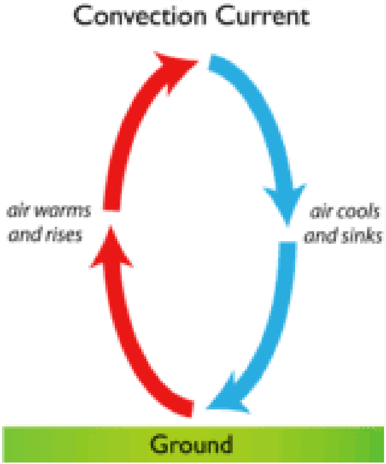

Figure 2: Convection in the atmosphere

- The Sun is primarily composed of plasma. We need a better framework to explain solar convection.

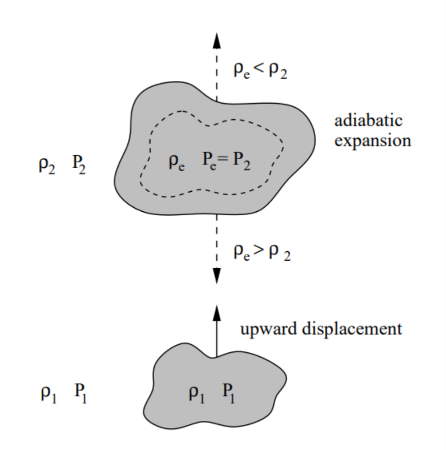

Figure 3: Schematic of solar convection

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- The gas parcel is initially in equilibrium with the ambient medium.
- The speed of upward transport is much less than the speed of sound in the medium. This ensures that any pressure fluctuations are quickly dissipated and throughout expansion, the pressure of the gas parcels equals the local pressure of the medium.
- The expansion is adiabatic, i.e, there is no heat exchange with the medium during expansion.

The Schwarzschild criterion

- At its new displaced position $r + \Delta r$, if $\rho_e > \rho_2$, the gas parcel is effectively heavier than its surroundings and thus sinks back down, quenching the perturbation. However, if $\rho_e > \rho_2$, the gas parcel is lighter than its surroundings and thus rises up, enhancing the perturbation.

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- For convective energy transport to be efficient, the gas parcel needs to travel a significant upward distance. This imposes some conditions on the density of the ambient medium.

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- For stability against convection (perturbations are quenched)

$$\rho_e > \rho_2$$
$$\delta\rho > \frac{d\rho}{dr}\delta r$$

The Schwarzschild criterion

- By definition,

$$\frac{\delta P}{P} = \gamma \frac{\delta \rho}{\rho}$$

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- Substituting this in the above inequality,

$$\frac{1}{\rho}\frac{d\rho}{dr} < \frac{1}{\gamma}\frac{1}{P}\frac{dP}{dr}$$

The Schwarzschild criterion

- The inequality flips while considering absolute values

$$\left|\frac{1}{\rho}\frac{d\rho}{dr}\right| > \left|\frac{1}{\gamma}\frac{1}{P}\frac{dP}{dr}\right|$$

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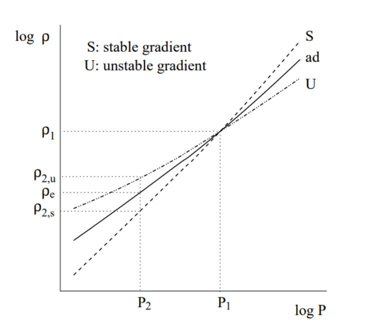

Figure 4: A plot of the density and pressure variations

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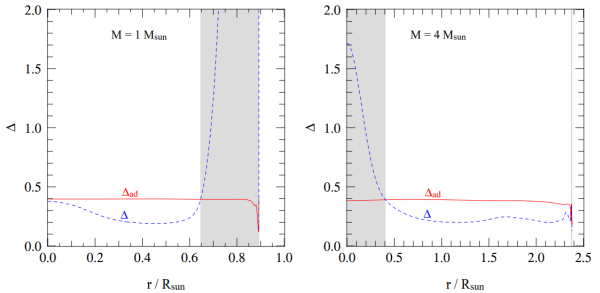

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$$S \approx \frac{R}{GM}\left(\frac{LR}{M}\right)^{2/3}$$

- Putting in the typical values, $S \approx 10^{-8}$ in the stellar core. Thus, convection is so efficient that even a tiny value of S is required to break the stability.

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- MLT assumes that convection is driven by the motions of gas blobs which are restricted to the z-direction, i.e, it assumes that there is no horizontal motion.
- Convection is driven by the energy released in the core which induces thermal fluctuations in the gas. Gas blobs which are relatively hotter than the ambient mean temperature, expand and thus rise. After rising a certain height (typically given by the local pressure scale height H_P), these gas blobs dissolve and dissipate their heat to the surroundings. This upward motion is adiabatic and the gas blobs are assumed to be in pressure equilibrium with their surroundings. Similarly, gas blobs cooler than the ambient mean temperature contract, become denser and move downward.

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- These rising and falling motions of the gas blobs maintain an energy flux which is directed radially outwards from the core.

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- Fluid motions are not just restricted to the one direction, but take place in the horizontal directions as well.
- Convection extends over many pressure and density scale heights and over a variety of length-scales.
- MLT is a local theory which predicts temperature fluctuations based on local conditions. In reality however, temperature fluctuations are mainly driven by non-local factors like radiative cooling at the surface.

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$$\frac{dm}{dt} = 0 \implies \frac{1}{\rho}\frac{d\rho}{dt} = -\frac{1}{V}\frac{dV}{dt}$$

- This equation shows that as a fluid parcel rises and becomes less dense, it has to correspondingly expand

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- Convection is typically pictured as warm upflows and cool downdrafts

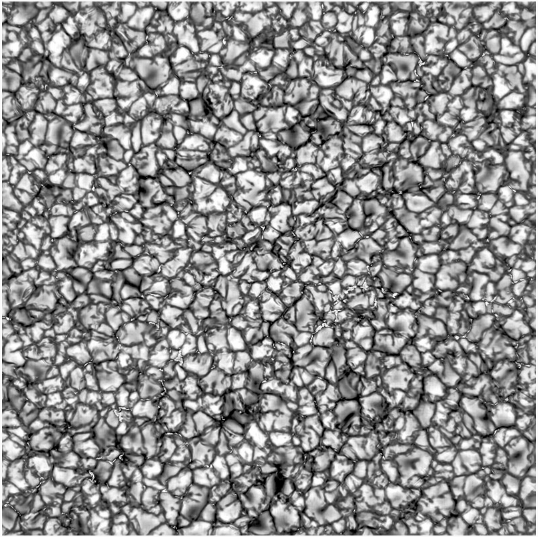

Figure 6: The granulation observed on the solar surface is a direct consequence of sub-surface convection. The bright granules represent warm upflows of gas while the dark intergranular lanes represent cool downdrafts.

References

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2. Nordlund et.al, 'Solar Surface Convection' - https://ui.adsabs.harvard.edu/abs/2009LRSP....6....2N/abstract