Investigating the Hubble Tension with Type Ia Supernovae

For Aakashganga & Science Club, IISER Pune

Anargha Mondal

Indian Institute of Science Education and Research, Pune

November 14, 2024

A Brief Introduction

A Theory for the Beginning

- While trying to build static universe models, Einstein introduced the cosmological constant.
- $\rho_{\text{vac}} = \frac{\Lambda}{8\pi G}$
- We interpret this as energy of the vacuum ($p = -\rho_{\text{vac}}$).
- Non-zero vacuum energies are expected in certain field theories, like zero-point fluctuations in quantum fields.

The ΛCDM model

- The standard cosmology is based on the well-known ΛCDM (cold dark matter) model.
- It relies on the existence of a cosmological constant Λ with an equation-of-state parameter $w = -1$ and a CDM component.
- This model is the most widely accepted paradigm to explain the structure and evolution of the late universe.

The Hubble Constant

- $H_0 = \frac{\dot{a}}{a}$
- Initial measurements of this value by Edwin Hubble[1]using Cepheids from the LMC seemed to imply that the **age of the universe was less than that expected for the solar system.**
- The measurement of H_0 improved from 10% uncertainty at the start of the 2000s to less than 2% by 2019.
- *Planck* 2018 in a flat ΛCDM model for the Hubble constant is $H_0 = 67.27 \pm 0.60\text{km s}^{-1}\ \text{Mpc}^{-1}$ at 68% CL[2]

[1] Hubble, 1929, *A relation between distance and velocity of extra-galactic nebulae*

[2] Planck Collaboration, Planck 2018 results. VI. Cosmological parameters

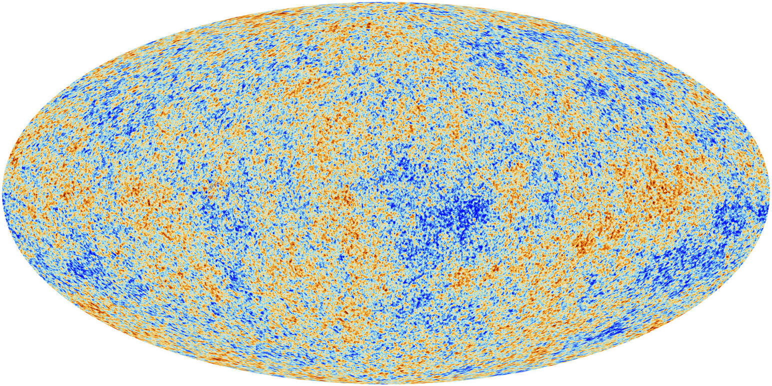


Figure: The Planck CMB map[3]

[3] ESA

Measurements

- Acoustic Peaks and Sound Horizon from the CMB
- Local measurements
  - Type Ia SNe
  - GRBs
  - Local Cepheids

A very powerful property that all Cepheids share:

$$\boxed{\mu_{th} = m - M = 5\log d_L + \text{constant}}$$

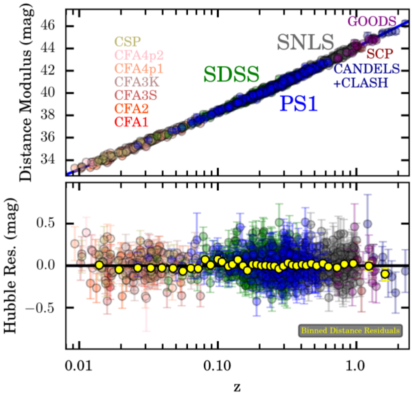


Figure: The Hubble Diagram for the Pantheon Sample of Type Ia Supernoave[4]

[4] Source: Scolnic et al, 2018

A Nobel Prize[5]

- The distances to the spectroscopic sample of SNe Ia measured by two methods are consistent with a **currently accelerating expansion** at confidence levels from 99.5% (2.8 σ) to more than 99.9% (3.9 σ) for $q_0 \equiv \frac{\Omega_M}{2} - \Omega_\Lambda$ using the prior that $\Omega_M \geq 0$.
- The data favor **eternal expansion** as the fate of the universe at the 99.7% (3.0 σ) to more than 99.9% (4.0 σ) confidence level from the spectroscopic SN Ia sample and the prior that $\Omega_m \geq 0$.
- The systematic uncertainties do not provide a convincing substitute for a positive cosmological constant.

[5] Source: Riess, A.G. The expansion of the Universe is faster than expected. Nat Rev Phys 2, 10–12 (2020)

Figure: Perlmutter (L), Riess (M), Schmidt (R)

Supernovae

- SNe arise from thermonuclear explosions of stars/star systems.
- Often outshine entire galaxies.

Figure: Holy Roman Emperor Henry III pointing up at a new star

- SNe are generally classified into two main categories according to spectroscopic features: Type I and Type II SNe[6]
- Type I SNe have **no hydrogen** (H) lines in their spectra, whereas **Type II SNe contain obvious H lines**.
- Type Ia SNe (SNe Ia) are a subclass of Type I, which exhibit strong singly ionized **silicon (Si) absorption** (Si II at 6150, 5800 and 4000 Å) features in their spectra.

[6] Minkowski 1941; Filippenko 1997; Parrent et al. 2014

Type Ia Supernovae

- SNe Ia are widely thought to be thermonuclear explosions of white dwarfs (WDs) in binary systems[7].
- B-band is about $M_B = -19.5$
- *Chandrasekhar limit*: A consequence of competition between **gravity and electron degeneracy pressure.**
- Predominantly, **Roche Lobe Overflows** and **Tidal Stripping** causes one of the binaries to collapse to supersede their Chandrashekhar limit.

[7] Hoyle & Fowler 1960

A Problem

All is not well with ΛCDM

- The ΛCDM is still the best and most elegant model we have to explain the Universe as a whole.
- An overview paper summarizing the cosmological legacy of the Planck mission (Planck Collaboration et al., 2020c) concluded: *The 6-parameter ΛCDM model continues to provide an excellent fit to the cosmic microwave background data at high and low redshift, describing the cosmological information in over a billion map pixels with just six parameters*.

The Hubble Tension

- *Planck Collaboration et al., 2020* decrees a Hubble constant of:

$$H_0 = 67.43 \pm 0.49 \text{kms}^{-1}\text{Mpc}^{-1}$$

- In contrast, *Riess et al., 2022* says that the latest value of the Hubble constant measured by the SH0ES collaboration based on Cepheid variables and Type Ia supernovae (SN) is:

$$H_0 = 73.01 \pm 0.99 \text{kms}^{-1}\text{Mpc}^{-1}$$

- There is a **huge degree of redundancy in the Planck** data and so there are many different ways in which the data can be partitioned, and there is a very low chance it is incorrect.
- On the other hand, **JWST photometry by the SH0ES team is in very good agreement with their earlier HST results**, effectively eliminating systematics associated with crowded field photometry as the source of the tension[8].
- It is therefore reasonable to conclude that either the **ΛCDM model is missing new physics** or the **SH0ES estimate is biased in some way**.

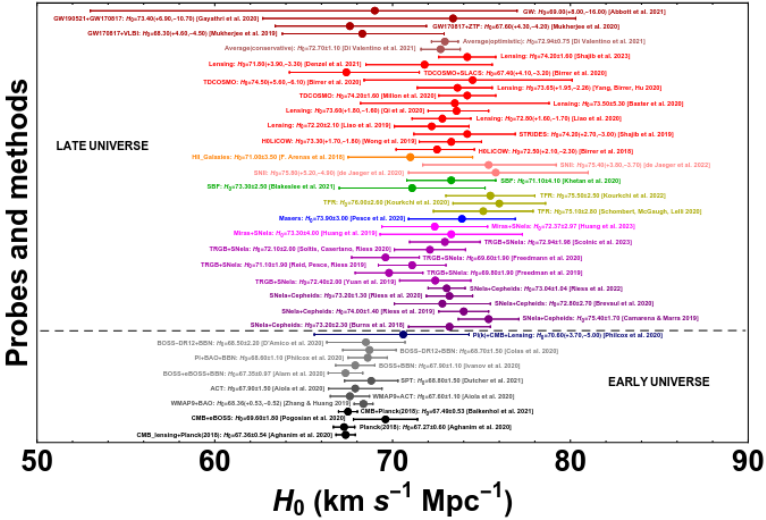
[8] Riess et al., 2023, 2024

Model	*Planck* TTTEEE	*Planck* TTTEEE+BAO
ΛCDM	67.44 ± 0.58	67.69 ± 0.42
ΛCDM+ m_ν	66.8 ± 1.2	67.8 ± 0.6
ΛCDM+ N_ν	66.4 ± 1.6	67.4 ± 1.2
ΛCDM+ $m_\nu + N_\nu$	$66.1^{+1.9}_{-1.6}$	67.5 ± 1.2
ΛCDM+ $m_{\rm str} + N_\nu$	67.1 ± 0.7	$67.89^{+0.45}_{-0.69}$
ΛCDM+ $n_{\rm run}$	67.25 ± 0.6	67.66 ± 0.45
ΛCDM+ Ω_k	56 ± 4	67.9 ± 0.7
ΛCDM+ $w_0 + w_a$	–	64.9 ± 2.1

Figure: Hubble Tension in extended ΛCDM models with 1σ errors[9]

No modifications reconcile this tension.

[9]Source: Efstathiou, Challenges to the ΛCDM Cosmology, 2024



For a more extensive record, refer to Di Valentino et al. In the Realm of the Hubble tension - a Review of Solutions.

Our Methodology

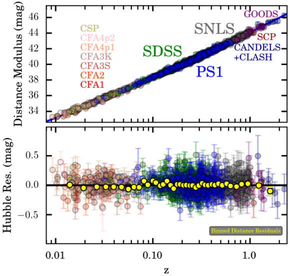
- We follow a similar methodology to Riess, Perlmutter, Scolnic and others in their 1998 breakthrough.
- To use SNe Ia as cosmological probes we have to consider their observed distance modulus, $\mu_{\rm obs}$, and compare with their theoretical distance modulus μ_{th}, defined as follows:

$$\mu_{th} = 5 \log d_L + 25$$

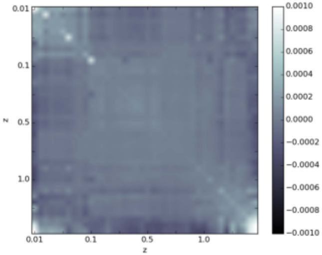
- In the current analysis, we make use of the Pantheon sample. This is a catalogue of 1048 SNe Ia with data from different surveys combined together in one collection. The $\mu_{\rm obs}$ can be obtained through the modified Tripp formula (Tripp 1998):

$$\mu_{\rm obs} = m_B - M + \alpha x_1 - \beta c + \Delta M + \Delta B$$

- In the Pantheon release, the absolute magnitude is fixed to $M = -19.35$ such that $H_0 = 70.0 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$.
- In our work, we have recalibrated to match the updated value of $H_0 = 73.04 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, from SH0ES Collboration., 2022.



(a) Hubble Diagram for Pantheon



(b) Covariance Matrix for Pantheon

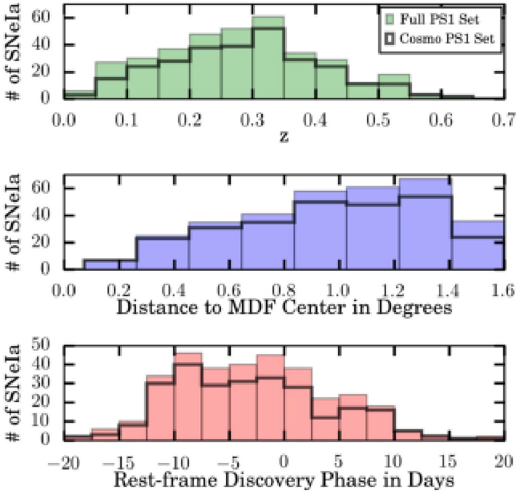


Figure: Histogram of Spectroscopically confirmed SNe

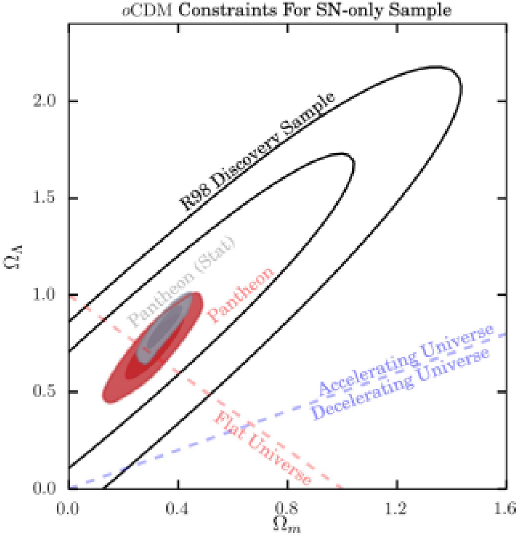


Figure: The R98 constraints vs Pantheon Constraints

Redshift Binned Analysis

- We define the distance residuals

$$\Delta\mu = \mu_{\text{obs}} - \mu_{\text{th}}$$

- $$\chi^2 = \Delta\mu^T \mathcal{C}^{-1} \Delta\mu$$

- $$\mathcal{C} = C_{\text{sys}} + D_{\text{stat}}$$

- D_{stat} is a diagonal matrix that includes the **total distance errors associated with every SN**. The latter takes into account the contributions from **photometric error**, **mass step correction**, **bias**, **peculiar velocity and redshift in quadrature**, **stochastic gravitational lensing**, and **intrinsic scatter**.

A Bit More on the Covariance Matrices...

- It is straightforward to build submatrices containing statistical contributions from each SN from D_{stat}, since D_{stat} is diagonal.
- However, the presence of the C_{sys} matrix, which is not diagonal, led us to write a customized code that extracts the submatrices, including also systematic errors.

Posterior and Likelihoods: MCMC Sampling

- We perform a Markov Chain Monte Carlo (MCMC)analysis using the D'Agostini method (D'Agostini 1994) to sample a posterior distribution and obtain the confidence intervals of the H_0 parameter at the 68% and 95% levels.
- MCMC algorithms generate a sample distributed according to the target distribution in a probabilistic fashion. The most common uses the **Metropolis-Hastings algorithm** as an example.

MCMC: Markov Chain Monte Carlo

- For exploring the **likelihood function** $\mathcal{L}$ or the **posterior distribution** p_{new} of the parameters, Markov chain Monte Carlo (MCMC) algorithms are today's method of choice when no functional expressions are available.
- It is based on a **random walk in the parameter space of the likelihood**, serially proposing new positions that are accepted or rejected according to its likelihood weights
- Assume we are interested in a probability density function $p(\theta)$ which is not given explicitly but can be calculated numerically up to a constant factor. If we want to learn about $p(\theta)$ we need to estimate it from a finite number of numerical evaluations.

Results

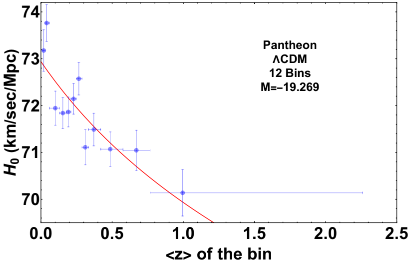
Evolution of the Hubble Constant with Redshift

- Our tests assume both the ΛCDM and the wCDM models, separately.
- $$w(z) = w_0 + w_a \frac{z}{1+z}$$
- For both the ΛCDM and the $w_0 w_a$CDM models, we set the following priors for MCMC: $60 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1} < H0 < 80 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$. Once we have obtained the values of H_0 for our bins, we perform a nonlinear fit of H_0 with the following functional form:

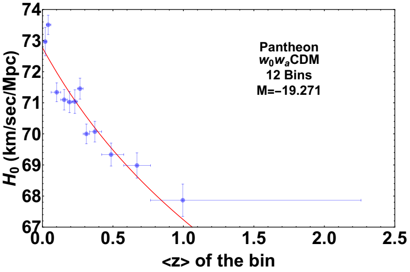
  $$H_0(z) = \frac{\tilde{H}}{(1+z)^{\alpha}}$$

  We here remark that the choice of this function is standard for characterizing the evolution of many astrophysical sources, and it is widely used for GRBs and quasars.

Equipopulation Binning Prescription



(a) ΛCDM



(b) $w_0 w_a$CDM

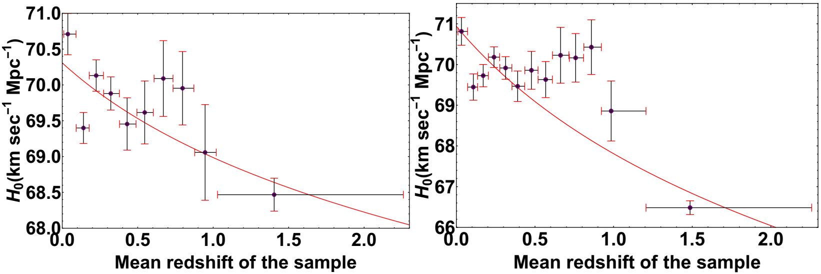
Figure: Pantheon 12 Bin prescription

Pantheon, Flat ΛCDM Model, Fixed Ω_{0m}			
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	α/σ_α
12	70.220 ± 0.179	0.016 ± 0.010	1.6

Pantheon, Flat w_0w_aCDM Model, Fixed Ω_{0m}			
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	α/σ_α
12	72.769 ± 0.191	0.114 ± 0.010	11.4

Figure: Table for equipopulation prescription

Equivolume Binning Prescription



(a) 10 bins

(b) 13 bins

Figure: Pantheon ΛCDM cosmology

Pantheon, Flat ΛCDM Model, Varying H_0			
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	α/σ_α
10	70.3071 ± 0.161727	0.0273713 ± 0.00523835	5.22517
13	70.9382 ± 0.163	0.0650801 ± 0.00426845	15.2468

Figure: Table for equivolume prescription

DES+Pantheon, Flat ΛCDM Model, Varying H_0			
Bins	$\tilde{H}_0$ (km s^{-1} Mpc^{-1})	α	α/σ_α
3	74.6628 ± 0.18689	0.290588 ± 0.00703825	41.287
4	67.9554 ± 0.0325176	0.0239669 ± 0.00167428	14.3147

Figure: Table for DES+Pantheon in ΛCDM cosmology

Discussion and Interpretation

Astrophysical Selection Biases

- The average **stellar ages and metallicities evolve with redshift**, so it may happen that the average corrected SN Ia brightness at higher redshift will be fainter than the one at lower redshift if the observed bias is caused by the progenitor age or metallicity[10]
- Sullivan et al. (2010) have suggested using host-galaxy mass as a third SN Ia brightness-correction parameter (after stretch and color), and this is done in Scolnic et al. (2018): **many of the associated systematic uncertainties of these effects are on the 1% level**
- Intergalactic dust extinction

[10] Childress et al. 2013

Theoretical Interpretation

- Local inhomogeneties: A perturbation in density causes a perturbation in the expansion rate[11].
  Lemaître-Tolmann-Bondi-like models?
- Modified gravity theories: $f(R)$ theories

[11] Kolb & Turner (1990), Marra et al. (2013), and Colgáin (2019)

f(R) theories

- Restates a more general Eisntein-Hilbert action to include some function f or the Ricci scalar.
- We hypothesise that the kinetic term of the Lagrangian contains information about the universe's total energy density. We suggest that this scenario can interpret our results, because the possibility of dealing with a significant universe acceleration requires a slow dynamics of the field, allowing that its potential term mimics a cosmological constant.
- The scalar field kinetic term, which contains second-order derivatives, is negligible and thus remains close to the ΛCDM model.
- $\phi(z) \approx (1+z)^{2\alpha}$ represents a scalar-field near-frozen dynamics, ensuring a very slow kinetic contribution to the universe energy density. This expression can be compared to our results.

Conclusions

- We find that there is a slow evolution of H_0 with redshift resulting from the fitting of various binning prescriptions.
- Although we considered a different number of bins, we obtain the same results for a decreasing $H_0(z)$ and with the evolutionary parameters α consistent with those cases of all bins.

- Interestingly, if the evolutionary pattern of $H_0(z)$ is extrapolated at the redshift of the most distant galaxy, $z = 11.09$, and of the last scattering surface, $z = 1100$, we obtain a value of $H_0(z)$ that is compatible within 1σ with the H_0 found by Planck in both the ΛCDM and the $w_0 w_a$CDM models, thus reducing the H_0 tension, albeit with larger errors.
- We have hence reduced the Hubble Tension to some respect, albeit, more surveys need to be done to reduce the error bars.
- Our results could highlight an intrinsic evolutionary behavior of $H_0(z)$: it is no longer a discrepancy between SNe Ia and Planck data, but an effect, in principle, observable at any redshift.
- Implications of a modification to the ΛCDM with near-frozen dynamics of a scalar field.