distinguishibility of states in Quantum Mechanics

Aashish Jaiswal

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step-0

- What is state of a system?

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- How do we know what state the system is in?

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- What is state of a system?
- How do we know what state the system is in?
- perform measurements!
- In classical mechanics you can know the state without disturbing the system.

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- $|\langle\alpha|\beta\rangle|$ can be thought of as the projection of $|\alpha\rangle$ on $|\beta\rangle$.

postulate-2: measurements

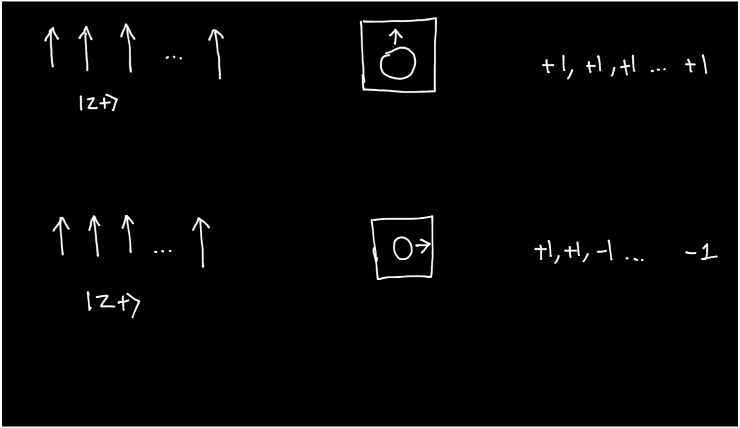

Figure: Given a collection of identical states we make some measurements of spin components along various axes. There is a pattern in averages.

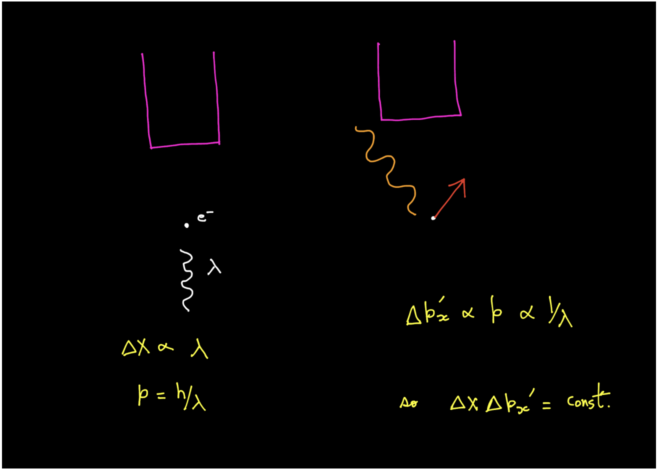

Figure: ΔX is the uncertainty in measuring the position of e^-. Classically you could've decrease λ to arbitrary small values. But for photons $p \sim \frac{1}{\lambda}$

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- $|\langle z+|z+\rangle|^2 = 1$. Given the state is $|z+\rangle$ we always get $|z+\rangle$ when we measure S_z.
- $|\langle x+|z+\rangle|^2 = |\langle x-|z+\rangle|^2 = 1/2$
- $|\langle z-|z+\rangle|^2 = 0$

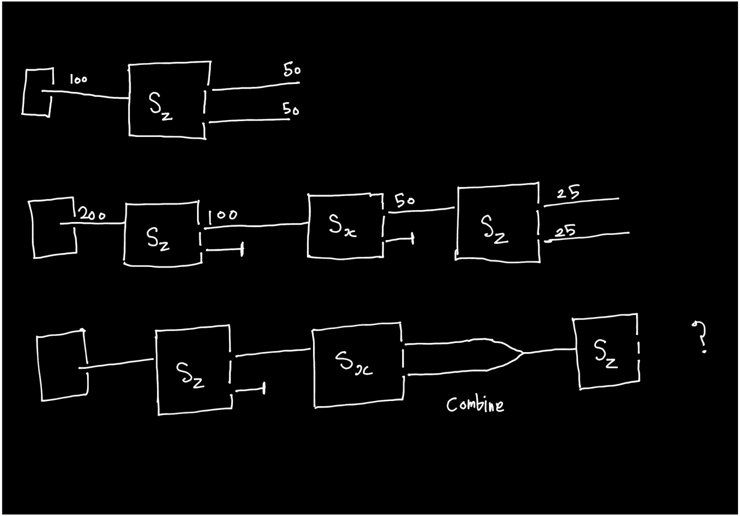

Figure: Stern gerlach experiments: sending **single** atoms through a non uniform B one at a time. Combining here means...

measurements

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- similarly one of the beam before combining can be tweaked (without performing any measurement) which changes it's phase to give the state

$$|z-\rangle = \frac{1}{2}(|x+\rangle - |x-\rangle)$$

distinguishability

- Can we distinguish two spins in $|z+\rangle$ and $|x+\rangle$ states when we don't know which spin is in which state? Can this be done reliably?

distinguishibility

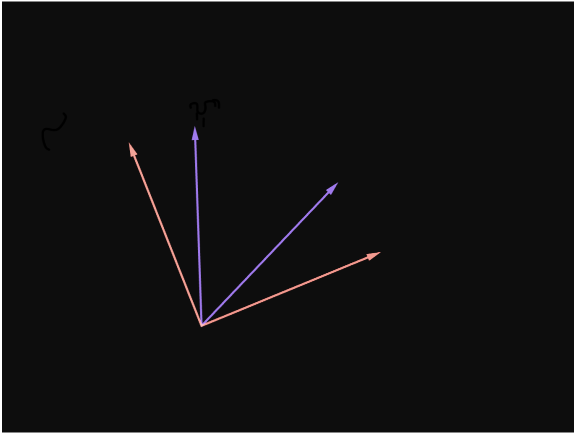

Figure: $|z+\rangle$ and $|x+\rangle$ in the state space can be imagined as shown. We take the orthogonal state on which $|z+\rangle$ and $|x+\rangle$ have biggest overlaps. $P_s = \cos^2(\pi/8) \approx 0.85$.

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- The overlap is preserved over time.

Alice, Bob and distinguishability

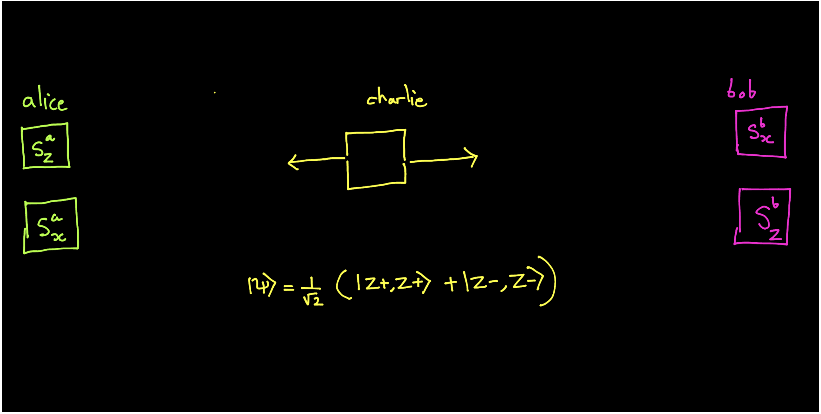

Figure: Charlie prepares two spins in an entangled state, sends one to Alice and Bob. They can make some measurements on their spins.

Can Alice send information to Bob instantaneously by making measurement on her spin?

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- Bob can then create large number of copies of his state. He can then measure S_z on all the copies of his state that he created.
- If the results are all $+1$ or all -1, then he can deduce that his spin state is $|z+\rangle$ and hence Alice measured S_z on her spin.

Cloning ?

But sadly cloning machines don't exist. They violate the unitarity of time evolution.

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- Standard measurements only tell us that there is a probability associated with each state corresponding to the given measurement outcome.
- We next highlight a measurement where, the measurement outcomes **either** tell the state with **complete certainty** or can't tell anything about the state.

Questions?

highlight of the procedure

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- We add an ancillary system in state $|s_o\rangle$ to our original system in an unknown state which could be either $|u\rangle$ or $|v\rangle$.
- It is possible to evolve the system + ancillia as follows: if the system is in $|u\rangle$ state, then

$$|u, s_o\rangle \longrightarrow p\,|u_1, s_1\rangle + q\,|r, s_2\rangle$$

if the system is in $|v\rangle$ state, then

$$|v, s_o\rangle \longrightarrow r\,|v_1, s_1\rangle + s\,|r, s_2\rangle$$

such that $\langle u_1|v_1\rangle = 0, \langle s_1|s_2\rangle = 0$.

why is such a time evolution possible

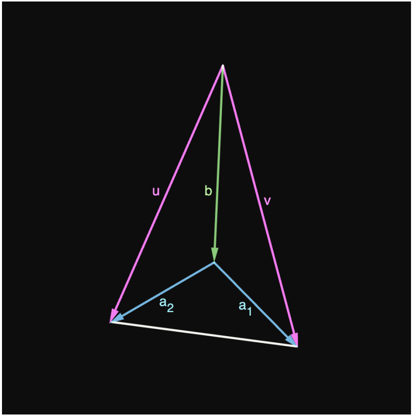

Figure: Time evolution can be thought of as a rotation in the state space. It always preserve the logical relations between the states.

Thank you!

references:

- The theoretical minimum: Quantum mechanics, Leonard Susskind.
  Both the lectures and his textbook are gems!
- Principles of Quantum Mechanics, R. Shankar
- Quantum computing and Quantum Information, Michael Neilsen and Isaac Chuang
- D. Dieks, "overlap and distinguishibility of quantum states", 1987, Physical Letters A, vol. 26