Supersymmetric CFT in 3D

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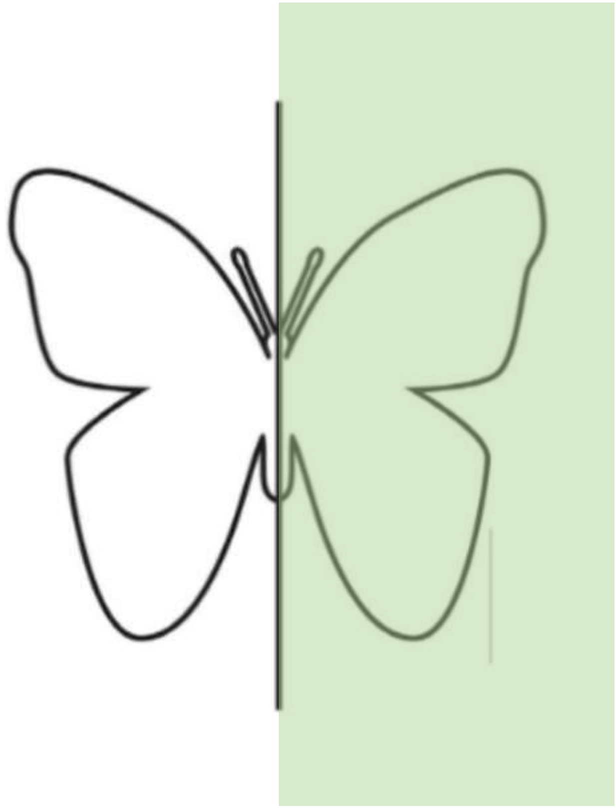
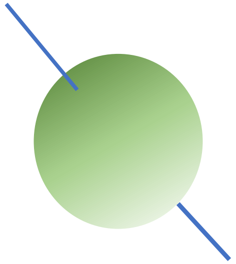
What's Symmetry?



Symmetry → Invariance

Invariance → Constraints on observables



→

$$E = \frac{L^2}{2I}$$

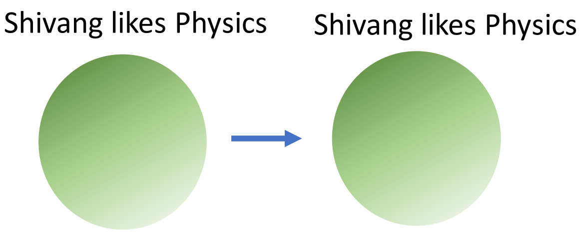
$$f(x, y, z) \equiv f(r)$$

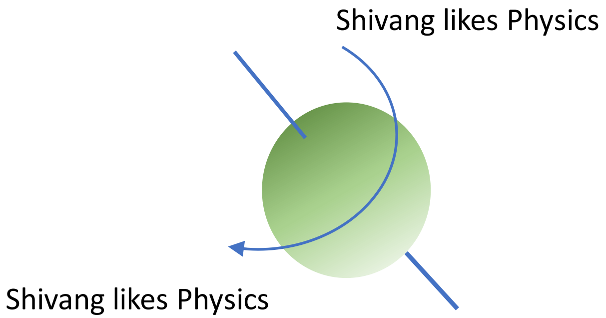
What is QFT?

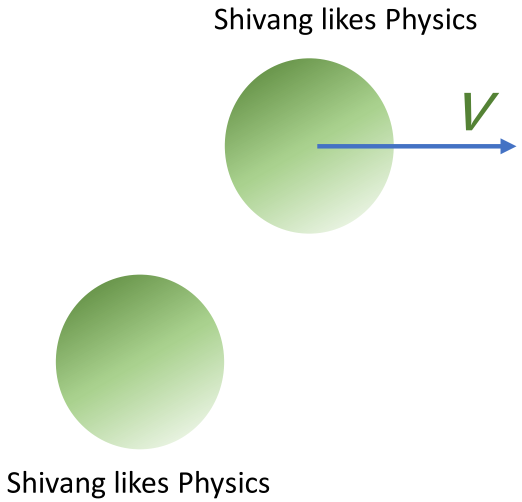
$$S = \int dt\, L(x, \dot{x}, t) \qquad S = \int d^{\mathrm{d}}x\, \mathcal{L}(x, \phi, \psi) \qquad S = \int d^4x \partial_\mu \bar{\phi} \partial^\mu \phi$$

What is QFT?

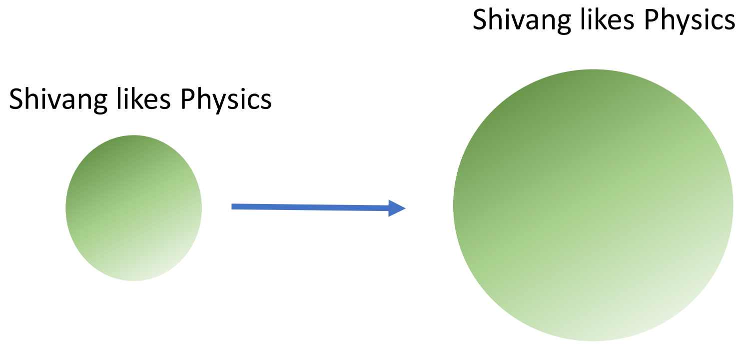
$$S = \int d^4x \partial_\mu \bar{\phi} \partial^\mu \phi$$







What is 'C'FT?



Special Conformal Transformation

Translation
Rotation
Relativistic Invariance
Dilatation
SCT

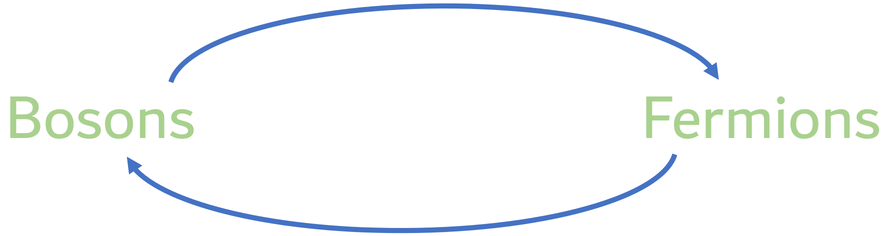
CONFORMAL FIELD THEORY

Now Supersymmetry

$$S = \int d^4x \, [\partial_\mu \bar{\phi} \partial^\mu \phi - i\bar{\psi} \not{\partial} \psi]$$

$$\delta S = \int d^4x \, \delta[\partial_\mu \bar{\phi} \partial^\mu \phi - i\bar{\psi} \not{\partial} \psi]$$

$$\delta\phi = \sqrt{2}\epsilon\psi \quad \text{and} \quad \delta\psi = \sqrt{2}i\sigma^\mu \bar{\epsilon}\, \partial_\mu \phi$$



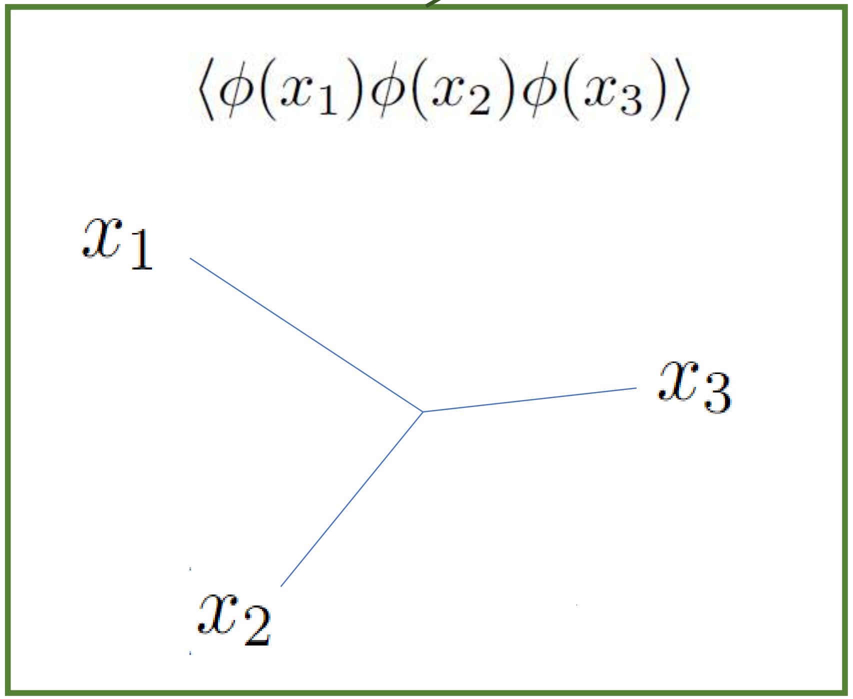
Supersymmetric CFT

$$S = \int d^4x \, [\partial_\mu \bar{\phi} \partial^\mu \phi - i \bar{\psi} \not{\partial} \psi]$$

$$\begin{aligned}
\Phi &= \phi + \theta_a \psi^a \\
\mathbf{O} &= \bar{\Phi}\Phi \\
&= \bar{\phi}\phi + \theta_a(\bar{\phi}\psi^a + \bar{\psi}^a \phi) + \theta^2 \bar{\psi}_a \psi^a \\
\mathbf{O} &= O_1 + \theta_a O^a_{1/2} + \theta^2 O_2 \\
\mathbf{J}^a_{1/2} &= O^a_{1/2} + \theta_b J^{ab} + \theta^2 F^a_{1/2}
\end{aligned}$$

What do I do?

To make a formalism to constrain CFT correlators using Supersymmetry in momentum space.



Two Point Function

$$\langle \mathbf{O}\mathbf{O} \rangle = \langle O_1 O_1 \rangle + \theta_{1m}\theta_{2n} \left\langle O^m_{1/2} O^n_{1/2} \right\rangle + \theta_1^2 \theta_2^2 \langle O_2 O_2 \rangle + \theta_1^2 \langle O_1 O_2 \rangle + \theta_2^2 \langle O_2 O_1 \rangle$$

$$\left\langle \mathbf{J}^a_{1/2} \mathbf{J}^b_{1/2} \right\rangle = \left\langle O^a_{1/2} O^b_{1/2} \right\rangle + \theta_{1m}\theta_{2n} \left\langle J^{am} J^{bn} \right\rangle + \theta_1^2 \theta_2^2 \left\langle F^a_{1/2} F^b_{1/2} \right\rangle + \theta_1^2 \left\langle F^a_{1/2} O^b_{1/2} \right\rangle + \theta_2^2 \left\langle O^a_{1/2} F^b_{1/2} \right\rangle$$

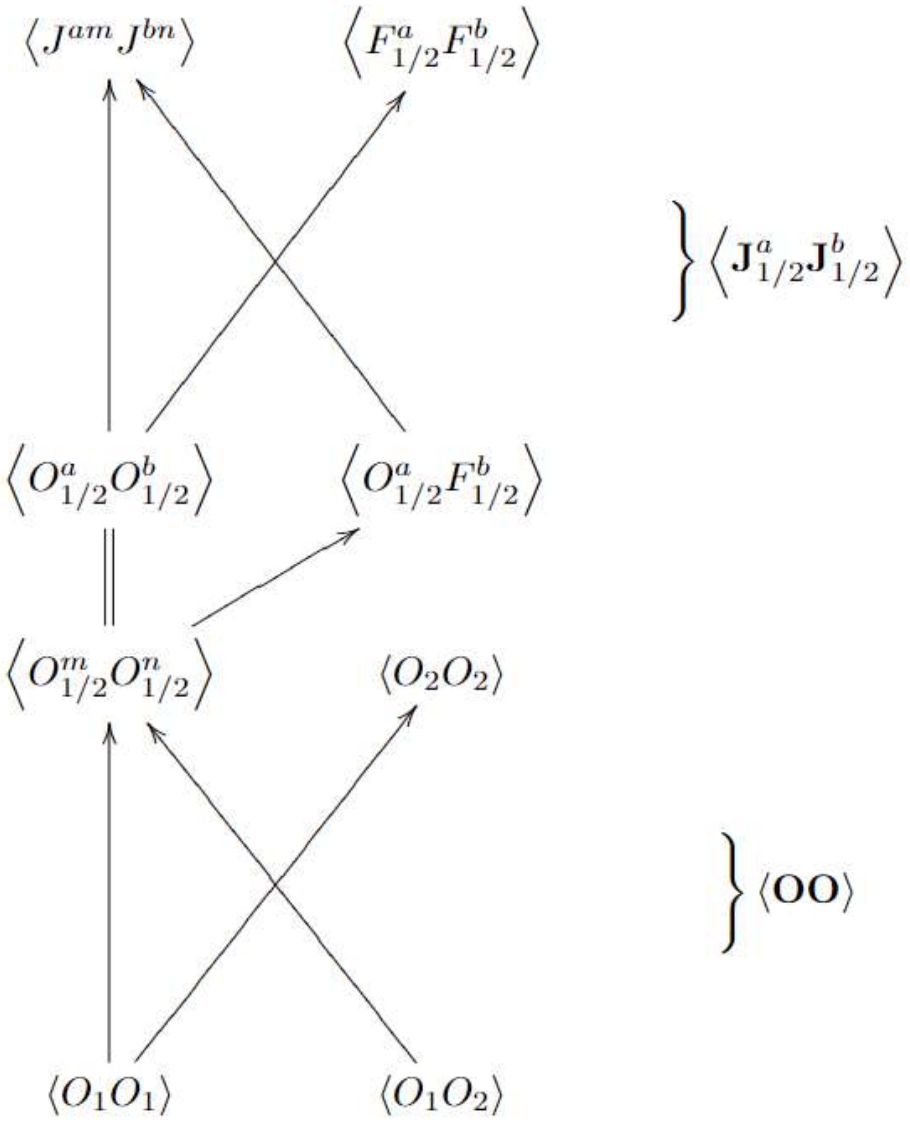
Pyramid Of Correlators

$$\langle \mathbf{OO} \rangle = \langle O_1 O_1 \rangle + \theta_{1m}\theta_{2n} \left\langle O^m_{1/2} O^n_{1/2} \right\rangle + \theta_1^2 \theta_2^2 \langle O_2 O_2 \rangle + \theta_1^2 \langle O_1 O_2 \rangle + \theta_2^2 \langle O_2 O_1 \rangle$$

$$\langle O_1(p_1) O_1(p_2) \rangle = A_1 = \frac{a_1}{p}, \quad \langle O_1(p_1) O_2(p_2) \rangle = A_2 = a_2$$

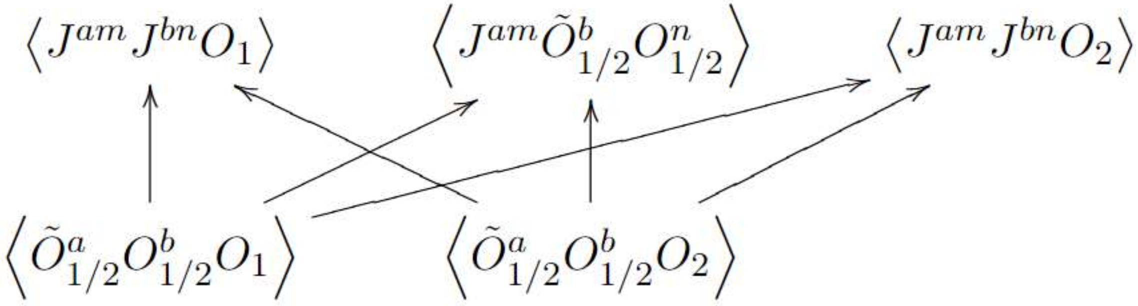
$$\left\langle O^m_{1/2}(p_1) O^n_{1/2}(p_2) \right\rangle = \frac{a_1}{2} \frac{\not{p}_1^{mn}}{p} - 2\epsilon^{mn} a_2,$$

$$\langle O_2(p_1) O_2(p_2) \rangle = a_1 \frac{p}{16}$$



Three Point Function

$$\left\langle \mathbf{J}^a_{1/2}\mathbf{J}^b_{1/2}\mathbf{O} \right\rangle$$



Scattering in 4D

$$\left\langle \mathbf{J}^-_{1/2}\mathbf{J}^-_{1/2}\mathbf{O}\right\rangle = (..)\left[\left\langle O^-_{1/2}O^-_{1/2}O_1\right\rangle + \frac{\left\langle O^-_{1/2}O^-_{1/2}O_2\right\rangle}{p_3}\right] + (..)\left[\left\langle J^-\tilde{O}^-_{1/2}O^-_{1/2}\right\rangle + \left\langle J^-\tilde{O}^-_{1/2}O^+_{1/2}\right\rangle\right] + (..)\left[\langle J^-J^-O_1\rangle + \frac{\langle J^-J^-O_2\rangle}{p_3}\right]$$

$$= (\dots)\frac{\langle 12\rangle}{\sqrt{p_1p_2}\,p_3E} \qquad + (\dots)\frac{\langle 12\rangle\langle 31\rangle}{\sqrt{p_2p_3}\,E^2} \qquad + (\dots)\frac{\langle 12\rangle^2}{p_3E^2}$$

Q?