Padé-Borel reconstructions of Euler-Heisenberg Lagrangians

Drishti Gupta

August 24, 2023

Outline

Background

- EH Lagrangian-Introduction
- Experimental evidence
- Schwinger Pair Production

Diving into the EHL

- Description of the system
- Form of the corrections
- Dyson's argument

Padé-Borel sums

- Motivation
- Padé-Approximants
- Calculating pair-production rates

Conclusion

EH Lagrangian

- The classical Maxwell equations develop corrections at short distances due to vacuum fluctuations.
- The classical Maxwell equations are **linear**, which means that classical EM fields can be added to each other without any ”interference terms”. This is not true in the quantum regime.
- The Euler-Heisenberg Lagrangian gives quantum corrections to the classical EM Lagrangian.
- The most significant prediction of the EH Lagrangian is Schwinger pair production: the spontaneous production of electron-positron pairs in high electric fields.

Experimental evidence I

JOURNAL ARTICLE

Evidence for vacuum birefringence from the first optical-polarimetry measurement of the isolated neutron star RX J1856.5-3754 FREE

R. P. Mignani, V. Testa, D. González Caniulef, R. Taverna, R. Turolla, S. Zane, K. Wu

Monthly Notices of the Royal Astronomical Society, Volume 465, Issue 1, 11 February 2017, Pages 492–500, https://doi.org/10.1093/mnras/stw2798

Published: 02 November 2016 **Article history** ▾

PDF | Split View | Cite | Permissions | Share ▾

Article | Published: 02 February 2022

Search for magnetic monopoles produced via the Schwinger mechanism

B. Acharya, J. Alexandre, P. Benes, B. Bergmann, S. Bertolucci, A. Bevan, H. Branzas, P. Burian, M. Campbell, Y. M. Cho, M. de Montigny, A. De Roeck, J. R. Ellis, M. El Sawy, M. Fairbairn, D. Felea, M. Frank, O. Gould, J. Hays, A. M. Hirt, D. L.-J. Ho, P. Q. Hung, J. Janecek, M. Kalliokoski, … O. Vives

+ Show authors

Nature **602**, 63–67 (2022) | Cite this article

Experimental Evidence II

REVIEW ARTICLE | JULY 29 2017

Electron-positron pair production in ultrastrong laser fields

Bai Song Xie; Zi Liang Li; Suo Tang

Check for updates

+ Author & Article Information

Matter and Radiation at Extremes 2, 225–242 (2017)

https://doi.org/10.1016/j.mre.2017.07.002 **Article history**

Access by Indian Inst

Vacuum birefringence by Compton backscattering through a strong field

Tobias N. Wistisen and Ulrik I. Uggerhøj

Phys. Rev. D **88**, 053009 – Published 17 September 2013

Article | References | Citing Articles (16) | PDF | HTML | Export Citation

ABSTRACT

We propose a novel scheme to measure nonlinear effects in electrodynamics arising from QED corrections. Our theoretical starting point is the Heisenberg-Euler-Schwinger effective Lagrangian

Before we start the math...

Some prerequisites to know:

1. Natural units: $c = 1$, $\hbar = 1$.
2. Summation conventions: $V^{\mu} = (V_0, V_1, V_2, V_3)$ represents a Lorentz vector (transforms as a vector under Lorentz transformations), $V_{\mu} = (V_0, -V_1, -V_2, -V_3)$ is the dual vector.
3. Repeated indices $V^{\mu}V_{\mu}$ means that the indices are being summed over.

Schwinger Pair Production

- The Dirac sea model- vacuum is made up of a sea of electrons with negative energy. Holes in the sea are interpreted as positrons.
- Imagine a potential barrier of height V. This potential barrier decreases the energy of the positive energy levels.
- When the potential barrier is of height $V \geqslant 2m$, the energy dips into negative energy levels and makes the possibility of pair production possible.

What is the system EHL describes?

- Imagine a **constant** electromagnetic field in vacuum.
- This electromagnetic field is produced by a source J^μ (made up of charge and currents). This source keeps the electromagnetic field constant no matter what.
- The Lagrangian density of such a system is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J^\mu A_\mu \tag{1}$$

  where $F^{\mu\nu}$ is the electromagnetic tensor. (check that this Lagrangian is gauge invariant)
- Using the EL equations gives us Maxwell's equations.

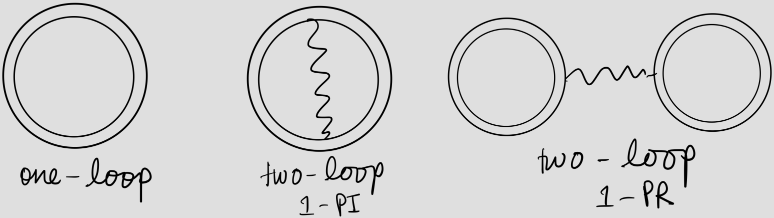
What do corrections look like?

- The corrections due to vacuum fluctuations are to the **source**, and not the field, since the value of the field is to be kept constant.
- The corresponding corrections to the Lagrangian is in terms of the gauge invariant quantity

$$f\left(F_{\mu\nu}F^{\mu\nu}\right) \tag{2}$$

- There are multiple orders of corrections. The n^{th} correction is called the n-loop correction.
- Each correction is of the order $\mathcal{O}(q^{2n})$, where q is the charge of the particle whose fluctuations the EM interacts with.

Feynman diagrams



Analytical nature of the corrections

Surprising
fact- the corrections to the
Lagrangian are **not analytic**!

Dyson predicted this.
His argument was that because
the behaviour in the electric
and magnetic field regime
is fundamentally different, the
corrections cannot be analytic.

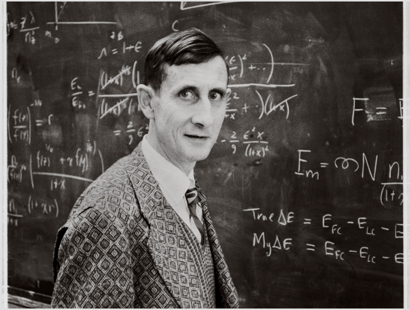


Figure: Freeman Dyson, 1923-2020

Consequence of non-analyticity

When one tries to expand the corrections about $B = 0$ (or $E = 0$), they find an expansion which doesn't converge **anywhere**! This is a problem, because in most cases, it is only possible to calculate the coefficients of the weak-field expansions and not the whole function. How do we get information from these coefficents that are part of a divergent series?

We use the technique of **Padé-Borel resummation**

Motivation

Suppose the correction has a weak-field expansion (for pure magnetic fields) of the form

$$f(B) \sim \sum_{n=0}^{\infty} a_n B^n \tag{3}$$

In most cases, a_n has a factorial divergence. Therefore, we define a new sum, called the **Borel sum**:

$$\hat{f}(B) = \sum_{n=0}^{\infty} \frac{a_n}{n!} B^n \tag{4}$$

This series now has a finite radius of convergence. We can recover the original function by doing a Laplace-like transform:

$$f(B) \sim \int_0^{\infty} dt \; e^{-t} \hat{f}(Bt) \tag{5}$$

Padé-Approximants

Now, we need a good function $\hat{f}$ which we can integrate over. $\hat{f}$ should have the desired weak-field expansion. For this, we use rational functions called Padé-approximants:

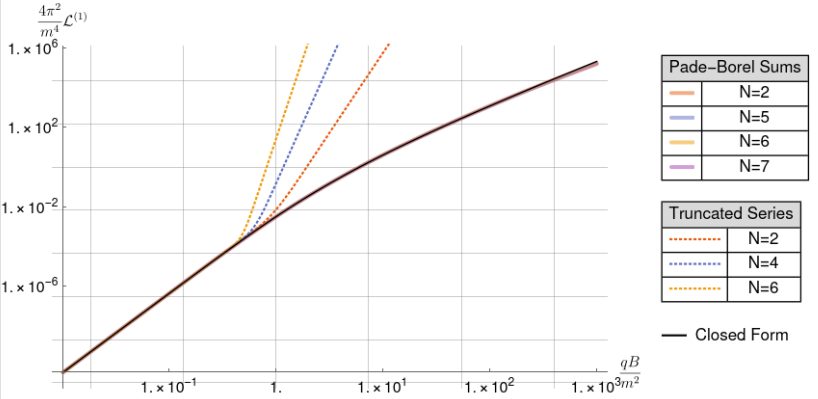
$$P_N^M(t) = \frac{a_0 + a_1 t + a_2 t^2 + \cdots + a_M t^M}{1 + b_1 t + b_2 t^2 + \ldots b_N t^N} \tag{6}$$

Padé approximants are awesome because they have the uncanny ability to provide good analytical continuations outside the domain of convergence. More the value of N and M, the more coefficients you need to construct it and better its accuracy

We substitute this function as $\hat{f}$ and get the **Padé-Borel resum**

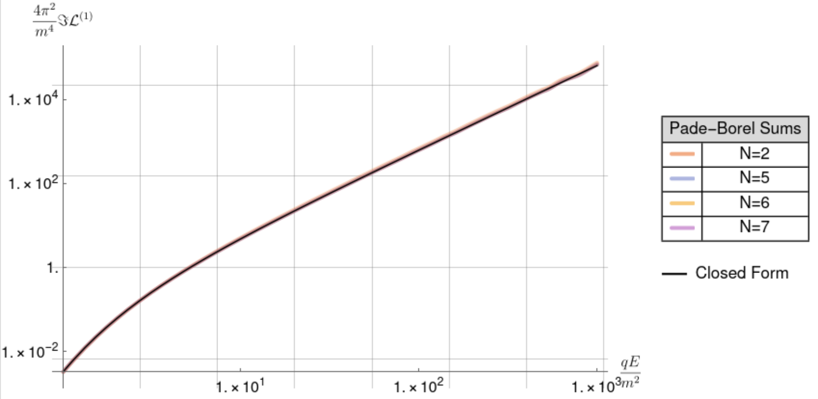
Do PB resummations work?

Yes! They work exceptionally well!



Pair production predictions

One of the major consequences of the non-analyticity of the EHL is that it obtains a non-trivial imaginary part in the electric field regime. There is no way we could've seen this in the weak-field expansion. However, the PB sum predicts the imaginary part accurately.



Conclusion

Divergent series are not bad, it means that there is weird physics hidden in there somewhere! PB resummations helps us extract this weird physics from unsummable series.